Department of Commerce

University of Calcutta

Study Material

Cum

Lecture Notes

Only for the Students of M.Com. (Semester II)-2020

University of Calcutta

(Internal Circulation)

Dear Students,

Hope you, your parents and other family members are safe and secured. We are going through a world-wide crisis that seriously affects not only the normal life and economy but also the teaching-learning process of our University and our department is not an exception.

As the lock-down is continuing and it is not possible to reach you face to face class room teaching. Keeping in mind the present situation, our esteemed teachers are trying their level best to reach you through providing study material cum lecture notes of different subjects. This material is not an exhaustive one though it is an indicative so that you can understand different topics of different subjects. We believe that it is not the alternative of direct teaching learning.

It is a gentle request you to circulate this material only to your friends those who are studying in Semester II (2020).

Stay safe and stay home.

Best wishes.

Paper CC 202:

Managerial Economics (ME)

CC 202 ME: Managerial Economics

Unit 2 : Demand Theories and Applications

THE THEORY OF CONSUMER BEHAVIOUR

UTILITY, MARGINAL UTILITY AND THEIR RELATIONSHIP

Utility means satisfaction. Thus utility can be defined as the satisfaction derived from the consumption of commodities or services. In other words, utility is the want satisfying power of a commodity. Suppose I want to write something on the blackboard. So this is my want and the chalk has the power to satisfy this want. Hence we can say that using chalk on the blackboard in order to write something on it provides me utility. Since the chalk has the power to satisfy the want, hence the chalk is providing utility. Utility is often termed as total utility (TU). The unit of utility is Util.

On the otherhand marginal utility (MU) is defined as the change in the total utility due to one unit change in the consumption of the commodity. So MU depicts the change in total utility due to additional consumption. Naturally marginal utility is diminishing with the consumption of the more units of the same commodity. For example, if I watch a popular movie on Monday, it gives me a considerable amount of satisfaction. However if I will watch the same movie on Tuesday also, my total utility will increase but the degree of satisfaction will definitely come down since it was already seen by myself earlier. Now if I repeatedly watch the movie everyday, total utility will increase at a diminishing rate which is suggestive of diminishing marginal utility. However, this will continue upto a certain level of consumption. One point will come when the consumption of the additional unit start to reduce the total utility. In such case, marginal utility is negative. The relationship between Total and Marginal utility can be represented through the following chart.

Units consumed	Total Utility	Marginal Utility
0	**0**	--
1	**12**	**12**
2	**21**	**9**

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3	**28**	**7**
4	**33**	**5**
5	**36**	**3**
6	**36**	**0**
7	**35**	**-1**

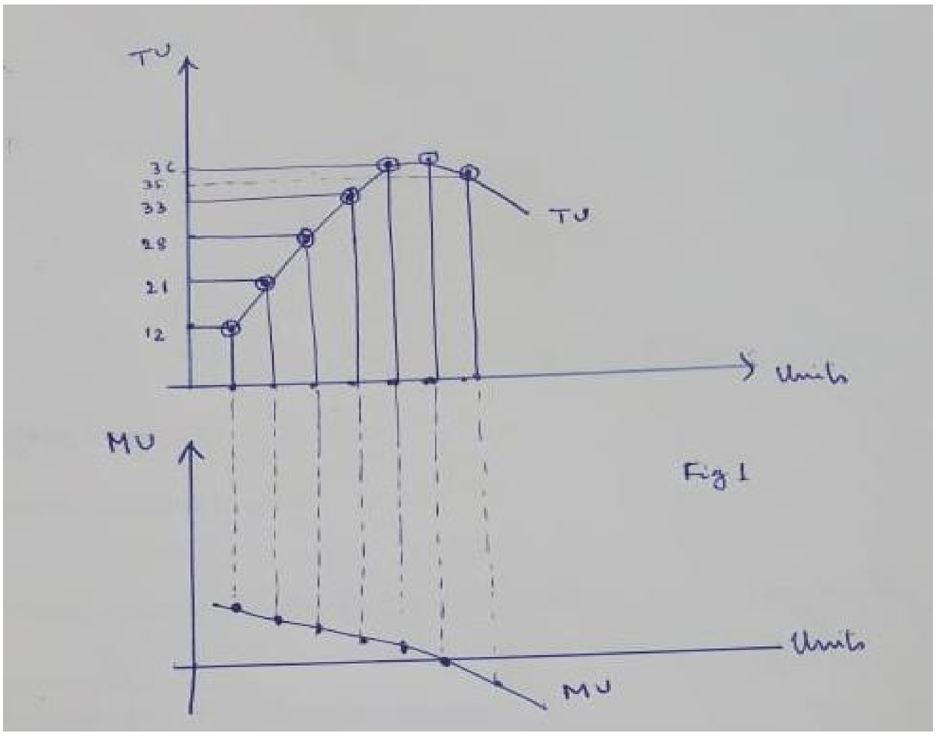
Thus from the table it is evident that marginal utility is initially positive and diminishing, then it touches zero and becomes negative thereafter.

We can represent this relationship through figure-1. After analysing this figure 1, we can identify the following relationship between TU and MU.

1. When TU rises, MU is positive i.e. > 0

2. When TU is maximum and constant, MU =0

3. When TU falls, MU is negative i.e. < 0



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DIFFERENT APPROACHES TO UTILITY

Generally there are two approaches to utility—cardinal and ordinal. Both of these approaches analyse utility in different manner. The cardinal utility theory says that utility is measurable just like prices and quantities. We can assign number of utils to each commodity in cardinal approach. In that sense it is a numerical and quantitative analysis. For example, if I say, that commodity 1 is providing me 20 utils of utility and commodity 2 is providing 25 utils of utility, then I am definitely following cardinal approach to utility. The economists who believed in cardinal approach, are of two types. One group believed that **utility is cardinal and additive.** Conversely the other group believed that **utility is cardinal but not additive.** Whatever may be the case, in cardinal approach both total utility and marginal utility is measurable.

In ordinal approach, the utility is not measurable like price and quantities. However we can order or rank the utility obtained from different commodities. Thus this approach is based on comparison and since here utility is not measurable, it is a qualitative analysis. For example if one says that one plate of biriyani is providing greater utility in comparison to one plate of fried rice, then obviously he is following ordinal approach to utility.

It should be noted that, in both of these approaches the assumption of diminishing marginal utility holds good.

Mathematically utility is a function of the quantity consumed of all the goods. Thus an utility function represents the mathematical relationship between the consumption of commodities and the utility obtained. It can be represented as—

$U = U(x_1, x_2)$

Where, U = Utility obtained

x_1 = units of commodity 1 consumed

x_2 = units of commodity 2 consumed

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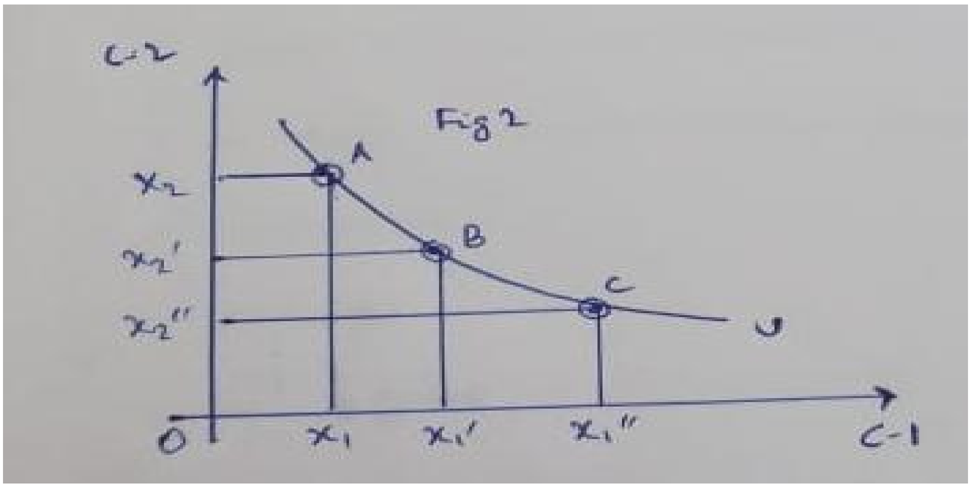
ORDINAL APPROACH AND INDIFFERENCE CURVE

The modern theory of consumer behaviour, based on ordinal utility uses the technique of indifference curve (IC) in order to analyse the utility. IC is the locus of points or commodity combinations in the commodity-commodity space which give same utility to the consumer. Thus along an indifference curve, utility remains the same. Since along an IC, utility remains the same, hence IC is also known as Iso-utility curve.

In figure 2, there are three combinations of C-1 and C-2 which are represented by point A (Ox_1, Ox_2), B ($Ox_1^{'}$, $Ox_2^{'}$) and C ($Ox_1^{''}$, $Ox_2^{''}$). Let all these combinations give same utility to the consumer. Now if we take locus of all these combinations or add all these combinations like A, B and C we will get a negatively sloped curve, which is known as indifference curve. Here the IC is given by U. Thus all the points on U represent same amount of utility or satisfaction.



To analyse the indifference curve in ordinal analysis we should rest upon the following assumptions—

1. The consumer is rational who always tries to maximise own utility.

2. We consider a 2-commodity world in order to avoid complications related to multi-dimensional space

3. Utility is ordinally measurable which implies the different utility levels can be arranged in an order

4. Different utility levels are interdependent.

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5. The transitivity relationship holds in case of indifference and preference. For instance if A is preferred to B and B is preferred to C, then as per transitivity A will be preferred to C.

6. More is Better, which implies that higher the consumption of the commodity, greater will be the utility obtained.

7. Consumer's income is given and it is equal to the consumer's expenditure

8. Marginal rate of substitution is diminishing

9. Prices of the commodities are given.

PROPERTIES OF INDIFFERENCE CURVE

1. IC is negatively sloped.

2. Two ICs can never intersect each other

3. Higher IC provides greater utility

4. IC is convex to the origin.

5. All the points on an IC are indifferent in terms of utility

The property 5 is directly coming from definition. So there is no need to prove it. However we will prove the first four properties.

<u>Property 1: IC is negatively sloped</u>

In figure 3 the IC is represented by U where all the points on it represent same amount of utility.

Let the utility function is given by

$U = U(x_1, x_2)$

Where, U = Utility obtained

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x_1 = units of commodity 1 consumed

x_2 = units of commodity 2 consumed

Now differentiating both sides totally we have

$dU = (\Delta U/\Delta x_1).dx_1 + (\Delta U/\Delta x_2).dx_2$

since along an IC the utility remains the same, hence dU =0

So, $(\Delta U/\Delta x_1).dx_1 = (\Delta U/\Delta x_1).dx_2$

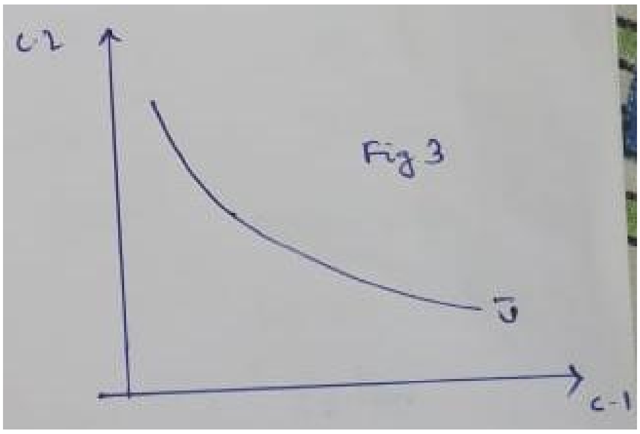
Or, $dx_2/ dx_1 = - (\Delta U/\Delta x_1)/ (\Delta U/\Delta x_2)$

i.e. the slope of the IC = $dx_2/ dx_1 = - (\Delta U/\Delta x_1)/ (\Delta U/\Delta x_2)$

Now, $(\Delta U/\Delta x_1)$ is nothing but the marginal utility of C -1 and $(\Delta U/\Delta x_2)$ is marginal utility of C-2.

Thus the slope of IC = $- (MU_1/ MU_2) < 0$

As both MU_1, $MU_2 > 0$ hence the slope is negative.



<u>Property 2: Two ICs can never intersect each other</u>

Let initially we assume two ICs U_1 and U_2 intersect each other at point A, which is given in figure 4. We select a point 'B' on IC U_1 and a point 'C' on IC U_2.

Initially when we consider U_1, we observe that both point A and point B are on the same IC. So as per definition we can say that point A and point B are indifferent in terms of utility. Similarly

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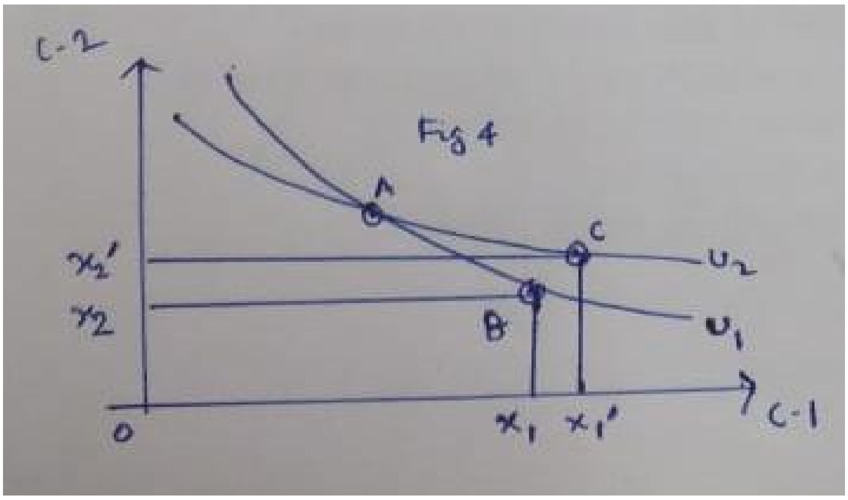
by considering IC U_2, we can say that point A and point C are same in terms of utility as per definition. So applying the rule of transitivity we can say that point B and C are indifferent in terms of utility as per definition. Thus according to the definition point B and point C provide same amount of utility.

Now if we look at the diagram, we observe at point B the consumer is able to consume Ox_1 units of C-1 and Ox_2 units of C -2, However at point C he can get Ox_1' (> Ox_1) units of C -1 and, Ox_2' (Ox_2) units of C-2. Thus point C involves higher consumption of both the commodities compared to point B. Since we know, more is better, according to the diagram we can conclude that point C should be preferred to point B as per utility is concerned.

Thus there arises a contradiction as diagram and definition are suggesting different result. This must be the result of our initial wrong assumption that two ICs can intersect each other.

So two ICs can never intersect each other.



Property 3: Higher IC provides greater utility:

Let there are two ICs—U_1 and U_2 among which U_2 is higher. These two ICs are represented in diagram 5. We select a point 'A' on U_1 and a point 'B' on U_2. At A, the consumer is able to consume Ox_1 units of C-1 and Ox_2 units of C -2, However at point B, he can get Ox_1' (> Ox_1) units of C -1 and, Ox_2' (Ox_2) units of C-2. Thus point B involves higher consumption of both the commodities compared to point A. Since we know, more is better, according to the diagram we can conclude that point B should be preferred to point A as per utility is concerned.

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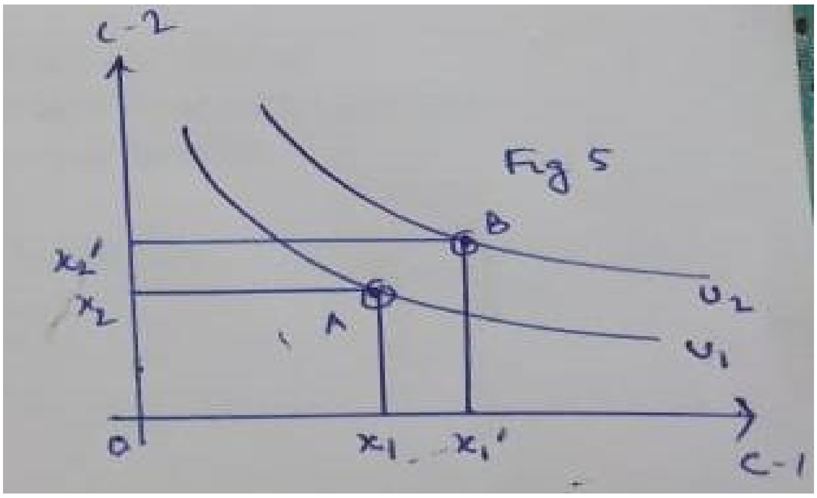
Now as point B is preferred to point A in terms of utility, all the points on U_2 are preferred to all the points on U_1 as per utility is concerned. Hence clearly U_2, which is the higher IC, depicts greater utility compared to U_1

Hence, higher IC provides greater utility.



Property 4: ICs are convex to the origin

A curve is said to be convex to the origin if a tangent drawn through any point on it is below the curve. Conversely in case of a curve which is concave to the origin, tangent drawn through any point of that curve should be over the curve.

However in order to prove the convexity of the IC, we should discuss the concept of Marginal Rate of Substitution (MRS) first. MRS measures the units of one commodity that has to be sacrificed in order to get one extra unit of the other commodity so that total utility remains constant. Now **if we can show that MRS is diminishing, then we can safely conclude that IC will be convex to the origin**.

In diagram 6, U is the relevant IC and the consumer initially stays at point 'A' where he is getting 1 unit of C-1 and Ox_2 units of C -2. Now in order to consume an additional unit of C-1, the consumer moves from A to B along the same IC where he can get 2 units of C -1 and Ox_2' units of C-2. Here—

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$MRS_{A\to B}$ =I $(Ox_2' - Ox_2)/(2-1)$I = I x_2x_2' I

Similarly if the consumer again wants to consume an additional unit of C-1, he will move from B to C along the same IC where we can get 3 units of C -1 and Ox_2'' units of C-2. Here—

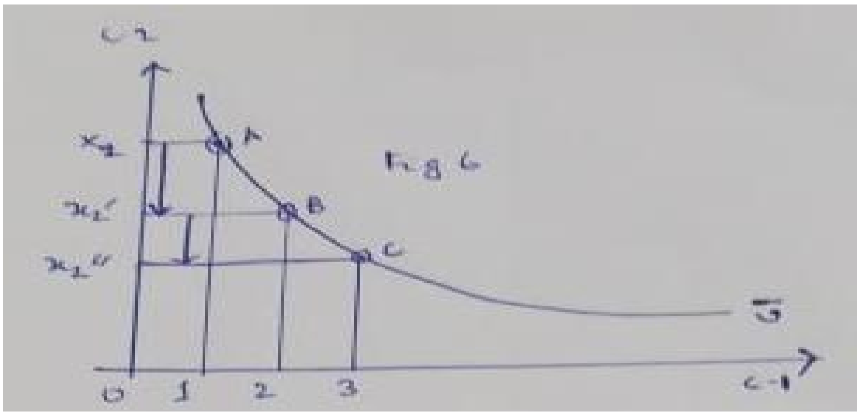
$MRS_{B\to C}$ =I $(Ox_2'' - Ox_2')/(2-1)$I = I $x_2'\ x_2''$ I

From the diagram, it is evident that I $x_2'\ x_2''$ I < I x_2x_2' I i.e. MRS is falling.

The diminishing MRS ensures the convexity of the IC.

So IC is convex to the origin.

It should be noted that, we can also show that when MRS is constant then IC will be a negatively sloped straight line. In addition to this, if MRS is increasing IC will be concave to the origin.



BUDGET LINE:

Budget line is the locus of the points or combinations of the commodities which represent same expenditure. Thus along the budget line consumer's expenditure is constant. In our analysis we assume that consumer's income is fixed and it is also equal to consumer's expenditure. Hence the budget line not only represents the fixed income or the expenditure of the consumer but also the equality between consumer's income and expenditure. Since the income of the consumer and the expenditure are equal, a consumer has to stay on the budget line. The consumer can not cross it or can not remain inside it. Any point above the budget line indicates the excees of expenditure over income. Conversely any point inside the budget lien indicates

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the exces of income over expenditure. In our analysis both of these abovementioned possibilities are assumed to be ruled out.

The budget line is a negatively sloped straight line.

Equation of a budget line

Let the consumer's fixed income is given by M. Let the consumer consumes two commodities—C-1 and C-2. Let the price of 1 unit of C-1 and 1 unit of C-2 are given by P_1 and P_2 respectively. We assume the consumer is consuming X_1 units of C-1 and X_2 units of C-2. Hence the total expenditure of the consumer is given by $(P_1X_1 + P_2X_2)$

Now since the budget line represents the equality between consumer's income and expenditure, the equation of the budget line is

$M = P_1X_1 + P_2X_2$

Construction of the budget line

Let consumer's income (M) = Rs 100, Price of C-1 (P_1) = Rs 5, Price of C-2 (P_2) = Rs 10.

Now here we will mention some of the combinations of C-1 and C-2 for which the expenditure and income are equal i.e. expenditure should be equal to 100.

C-1	C-2
20	0
0	10
10	5
8	6
6	7
4	8
12	4

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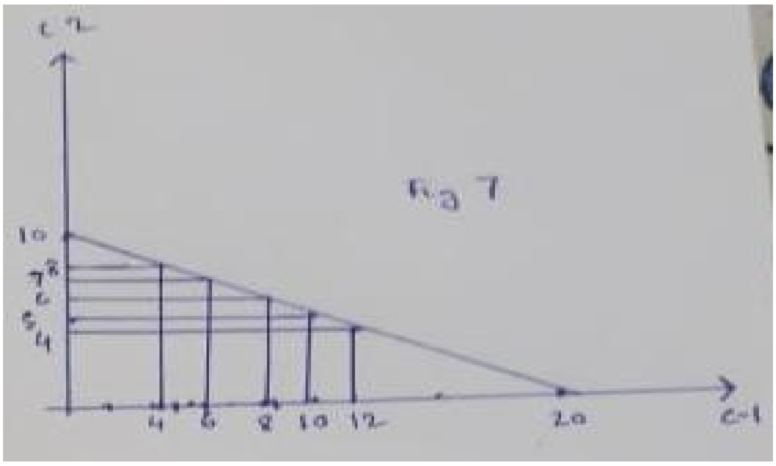
All the above mentioned combinations of C-1 and C-2 are different but they are similar in one respect. All of these combinations are suggestive of same expenditure i.e. 100. If we add all these combinations viz. (20, 0), (0, 20), (10, 5), (8, 6), (6, 7), (4, 8), and (12, 4) then we will get the budget line which is represented in figure 7.



Slope and the intercept of the budget line

Let the budget line in figure 8 is given by AB.

The slope of the budget line is tan θ = OA/OB..........(1)

Now if the consumer stays at point A, he can consume zero unit (0) of C-1 and OA units of C-2. So his budget equation becomes-

$M = P_1.0 + P_2. OA$

So, $OA= M/P_2$............(2)

Conversely if the consumer stays at point B, he can consume OB units of C-1 and zero (0) unit of C-2. Then his budget equation becomes—

$M = P_1. OB + P_2.0$

So, $OB = M/P_1$.........(3)

From (1), the slope of the budget line = $OA/OB = - (M/P_2)/ (M/P_1)$

$= - P_1/P_2$

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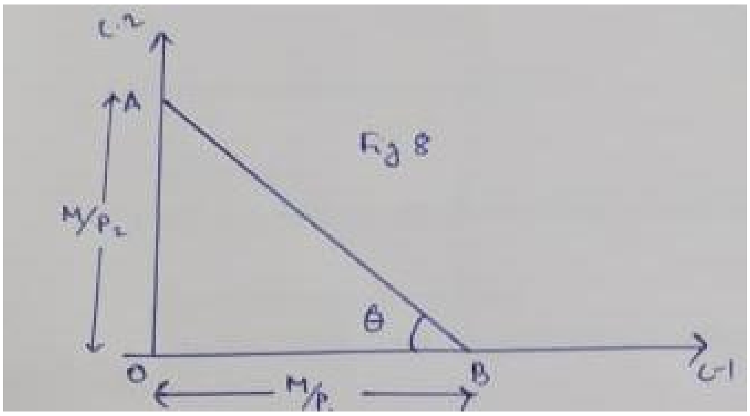
Here the negative sign is inserted before the expression as there exists an inverse relationship between the consumption of C-1 and C-2.

So slope of the budget line is $(-P_1/P_2) = |P_1/P_2|$



We can also determine the slope of the budget line mathematically.

We know the budget equation is given by—

$M = P_1X_1 + P_2X_2$

Or, $P_2X_2 = M - P_1X_1$

Or, $X_2 = (M - P_1X_1)/ P_2$

Or, $X_2 = M/P_2 - (P_1/P_2). X_1$

Or, $X_2 = - (P_1/P_2). X_1 + M/P_2$

This is the equation of a straight line which is in the form of y=mx + c, whose slope

(m) = $- (P_1/P_2)$ and vertical intercept (c) = M/ P_2

So the slope of the budget line is given by $(-P_1/P_2) = |P_1/P_2|$

The vertical intercept (OA) = M/ P_2

The horizontal intercept (OB) = M/ P_1

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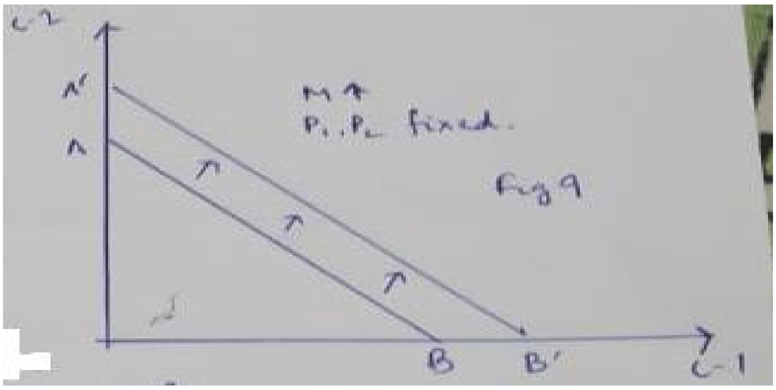
Shift and rotation of the budget line

The budget equation is given by

$M = P_1X_1 + P_2X_2$

Here, income and prices are constant, however the combinations of commodities vary. Now if there is a change in the income level or the prices the budget line will either shift or rotate. If there is no common point between the old and the new budget line then we can term it as the shift, where as if there is a common point between the old and the new budget line, then we can call it as the rotation of the budget line.

In figure 9, income (M) changes and prices i.e. P_1 and P_2 remain same. Thus both the vertical and horizontal intercepts M/P_2 and M/P_1 will rise, which implies an upward shift of the budget line. Now we know slope of the budget line is given by = $|P_1/P_2|$= Modulus P_1/P_2. Since here both prices remain same, the slope will be unchanged. **This suggests that if M increases and P_1 and P_2 remain same, then there will be an upward parallel shift of the budget line from AB to A'B'.**



In figure 10, we assume income (M) and price of C-2 i.e. P_2 remain fixed but there is a fall in the price of C-1 i.e. P_1.

This implies the vertical intercept (M/P_2) remains same, however the horizontal intercept (M/P_1) rises due to the fall in the denominator part i.e. P_1. **So here the old budget line AB will rotate rightward to AC.**

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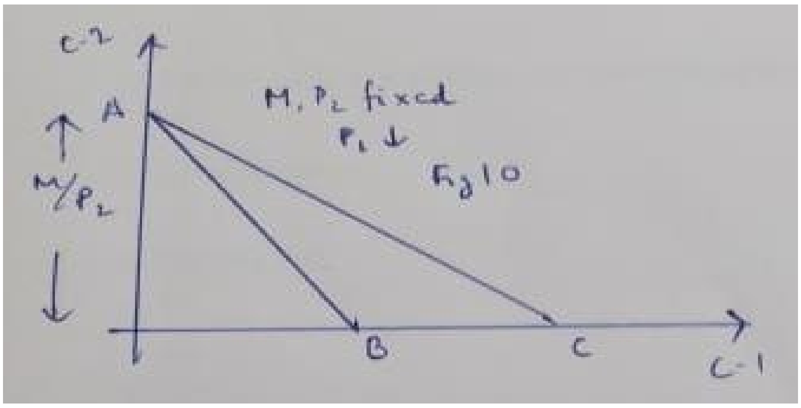
We can consider various rotations and shifts of budget line by altering the conditions.



CONSUMER EQUILIBRIUM

A consumer is said to be in equilibrium if he attains maximum possible satisfaction or utility from his purchases of commodities subject to the budget constraint. Thus a consumer always tries to maximise utility with his given income. We know indifference curve (IC) indicates the utility level and the budget line represents the fixed income of the consumer. The objective of the consumer is to reach the highest possible IC remaining on the budget line. Here we assume that

In figure 11, U_0, U_1, U_2, U_3 and U_4 are five representative ICs among which U_4 is the highest. The consumer's fixed income is given by the budget line AB. We know the consumer has to stay on the budget line. Now, among the representative ICs, the consumer can not reach U_3 and U_4, since they are beyond his budget line. Thus he has to choose among the points K, L, M, N and E. It is clear from the diagram that the consumer will definitely choose point E as it is on a relatively higher IC U_2, compared to K or L (on U_0) and M or N (on U_1). So in this circumstance point E provides the consumer with maximum possible utility. So point E is known as consumer's equilibrium point.

We can easily identify the conditions of consumer's equilibrium. In the diagram IC U_2 is drawn in such a way so that the budget line AB is tangent to it at point E. Thus the slope of IC U_2 at E can be measured by the slope of the tangent AB through E. Here the budget line is acting as the tangent. Thus the conditions of equilibrium can be given as-

At the point of equilibrium,

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1. Slope of IC = Slope of budget line

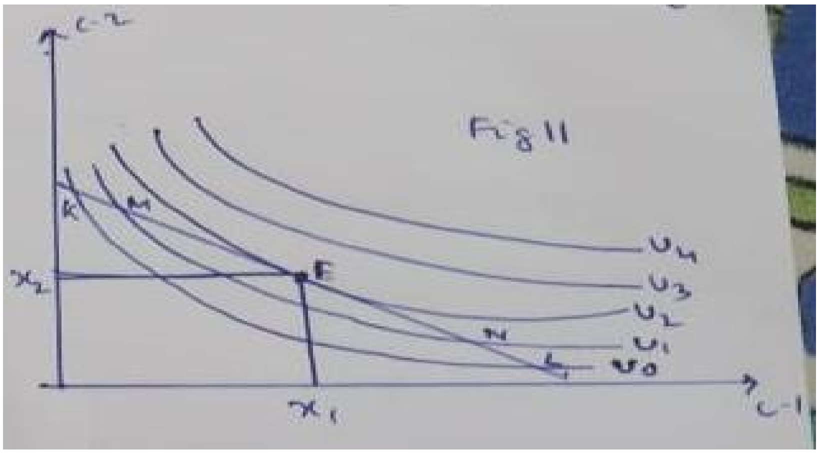
Or, $-(MU_1/MU_2) = -(P_1/P_2)$

Or, $MU_1/P_1 = MU_2/P_2$

i.e utility of Re 1 worth of C-1 is equal to utility of Re 1 worth of C-2 at the point of equilibrium.

2. Since both C-1 and C-2 are good commodities, the relevant IC should be convex to the origin.

As point E refers to the point of equilibrium of the consumer, it can be understood from the diagram that if the consumer purchases Ox_1 units of C-1 and Ox_2 units of C-2 then he will be in the equilibrium. Ox_1 units of C-1 and Ox_2 units of C-2 are known as the optimal purchases or the optimal demand functions of the consumer.



ECONOMIC INTERPRETATION OF THE CONDITION OF CONSUMER EQUILIBRIUM

Here we will discuss the economic interpretation of the consumer's equilibrium condition. Here we will show why point E can be termed as the equilibrium point. For this we select two different points M and N to the left and right of point E respectively on the same budget line AB. This is depicted in figure 12.

At point M, the slope of IC can be measured by the slope of the tangent ST through it. Thus at M, slope of IC (STO) > slope of budget line (ABO). So at M, $(MU_1/MU_2) > (P_1/P_2)$

Or, $MU_1/P_1 > MU_2/P_2$

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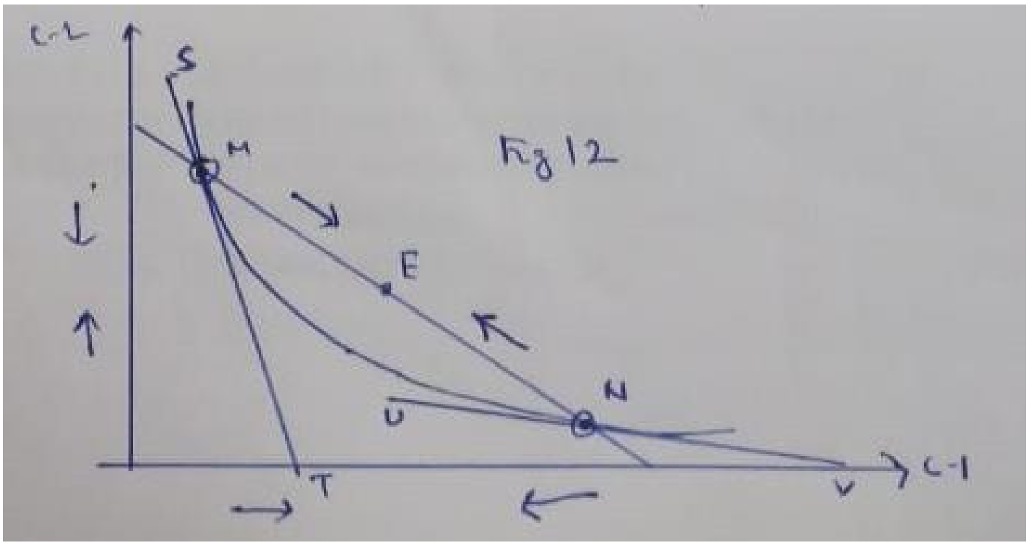
Hence at M, the utility of Re 1 worth of C-1 is greater than that of C-2. So here the consumer will try to consume more of C-1 at the expense of C-2 in order to raise the utility level. Thus here the consumer will move down right towards E along the same budget line AB in order to get more C-1 by sacrificing C-2.

Conversely, At point N, the slope of IC can be measured by the slope of the tangent UV through it. Thus at N, slope of IC (UVO) < slope of budget line (ABO). So at N, $(MU_1/ MU_2) < (P_1/P_2)$

Or, $MU_1/ P_1 < MU_2/P_2$

Hence at N, the utility of Re 1 worth of C-2 is greater than that of C-1. So here the consumer will try to consume more of C-2 at the expense of C-1 in order to raise the utility level. Thus here the consumer will move up left towards E along the same budget line AB in order to get more C-2 by sacrificing C-1.

Finally he will settle at point E, where the slope of IC and the slope of budget line are equal, i.e. utility of Re 1 worth of C-1 is equal to the utility of Re 1 worth of C-2. Thus the point of equilibrium is nothing but a state of rest. At E, since the consumer is getting same amount of utility by spending Re 1 on both the commodities, there will be no tendency from the part of him to shift from that point. Hence point E is the equilibrium point of the consumer.



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CC 202 ME: Managerial Economics

MODULE—I

[Prepared by-Dr. Titas Kumar Bandopadhyay]

Constrained optimization problem:

This study material tries to show how to solve the constrained optimization problem. In the case of constrained optimization problem, we have to maximize/minimize the objective function subject to the given constraint.

Suppose, our problem is to maximize $Y = f\left(x_1, x_2\right)$ **subject to** $K = a_1 x_1 + a_2 x_2$**, where** K, a_1, a_2 **are constant. We can solve the problem by using the Lagrangean method. Actually, the Lagrangean method converts the constrained optimization problem into an unconstrained one.**

The Lagrangean expression is

$$L = f\left(x_1, x_2\right) + \lambda\left[K - a_1 x_1 - a_2 x_2\right]$$

Where $\lambda > 0$ and λ **is the Lagrange Multiplier.**

The first order conditions for this optimization problem are

$$L_1 = \frac{\partial L}{\partial x_1} = f_1 - \lambda a_1 = 0 \quad \textbf{(1)}$$

$$L_2 = \frac{\partial L}{\partial x_2} = f_2 - \lambda a_2 = 0 \quad \textbf{(2)}$$

$$L_\lambda = \frac{\partial L}{\partial \lambda} = K - a_1 x_1 - a_2 x_2 = 0 \quad \textbf{(3)}$$

From (1) and (2) we get,

$$\frac{f_1}{a_1} = \frac{f_2}{a_2} \quad \textbf{(4)}$$

From (3) we get,

$$K = a_1 x_1 + a_2 x_2 \quad \textbf{(5)}$$

Solving (4) and (5) we get the optimum values of x_1, x_2 **as**

$$x_1^* = x_1^*\left(a_1, a_2, K\right)$$
$$x_2^* = x_2^*\left(a_{1,}, a_2, K\right)$$

The second Order Condition for Y maximization is $\left|\bar{H}\right| > 0$. **That means the value of the bordered Hessian Determinant is greater than zero.**

Now, $\left|\bar{H}\right| = \begin{vmatrix} L_{11} & L_{21} & L_{a1} \\ L_{12} & L_{22} & L_{a2} \\ L_{1a} & L_{2a} & L_{aa} \end{vmatrix}$

Using Equations (1),(2), (3) we can write,

$\left|\bar{H}\right| = \begin{vmatrix} f_{11} & f_{21} & -a_1 \\ f_{12} & f_{22} & -a_2 \\ -a_1 & -a_2 & 0 \end{vmatrix} > 0$ **(all second partials are evaluated at the optimum values of choice variables)**

So, the optimum values of the choice variables give the maximum values of the objective function.

Note that for maximization problem the Bordered Hessian Determinant must be negative.

Now, let us solve a problem.

Max. $\begin{array}{l} U = xy \\ S.T.M = xP_x + yP_y \end{array}$

The Lagrangean expression is $L = xy + \text{a}\left[M - xP_x - yP_y\right]$

where $\text{a} > 0$ **and it is the Lagrange multiplier.**

The F.O.C.s are:

$L_x = y - \text{a}P_x = 0$ **(1)**

$L_y = x - a P_y = 0$ **(2)**

$L_a = M - xP_x - yP_y = 0$ **(3)**

From (1) and (2) we get,

$\frac{y}{P_x} = \frac{x}{P_y} \Rightarrow xP_x = yP_y$ **(4)**

Using (3) and (4) we get,

$$x^* = \frac{M}{2P_x}$$

$$y^* = \frac{M}{2P_y}$$

These are the demand functions for x and y.

The S.O.C. is $\left|\bar{H}\right| > 0.$

Here, $\left|\bar{H}\right| = \begin{vmatrix} L_{xx} & L_{yx} & L_{ax} \\ L_{xy} & L_{yy} & L_{ay} \\ L_{xa} & L_{ya} & L_{aa} \end{vmatrix} = \begin{vmatrix} 0 & 1 & -P_x \\ 1 & 0 & -P_y \\ -P_x & -P_y & 0 \end{vmatrix} = 2P_xP_y > 0$ **(at the optimum values of x and y)**

Thus the SOC is satisfied and the optimum level of purchase of the two goods are

$$x^* = \frac{M}{2P_x}, y^* = \frac{M}{2P_y}$$

Remarks:

1.

$$x^* = \frac{M}{2P_x}, y^* = \frac{M}{2P_y}$$

$$\Rightarrow \frac{\partial x^*}{\partial P_x} = -\frac{M}{2P_x^2} < 0$$

$$\frac{\partial y^*}{\partial P_y} = -\frac{M}{2P_y^2} < 0$$

This implies that the demand curves are negatively sloped.

2. The price elasticity of demand is unit for both of the two goods. This implies that the demand curves are rectangular hyperbolic.

Home Work:

Given $U = (x+2)(y+1), P_x = 2, P_y = 5, M = 51.$

a. Write the Lagrangean function.

b. Find the optimal levels of purchase of x and y.

c. Is the second-order condition for maximum fulfilled?

CC 202 ME: Managerial Economics

Module –I

Unit: 3 Production and Cost

(Prepared by Dr. Mahananda Kanjilal and Anindita Basu)

Prodⁿ —

Prod^n

C. D. Prod^n f^n

$$X = b_0 L^{b_1} K^{b_2}$$

$$= b_0 (\lambda L)^{b_1} (\lambda K)^{b_2}$$

$$= b_0 \lambda^{(b_1+b_2)} L^{b_1} K^{b_2}$$

In C.D. $b_1 + b_2 = 1$

$$= \lambda X .$$

1) $MP_L = \frac{\partial X}{\partial L} = b_1 b_0 L^{b_1 - 1} K^{b_2}$

$$= b_1 (b_0 L^{b_1} K^{b_2}) L^{-1}$$

$$= b_1 \left(\frac{X}{L}\right) = b_1 (AP_L)$$

$$MP_K = b_2 \frac{X}{K} = b_2 (AP_K)$$

2) MPS =

$$MRS_{L,K} = \frac{\partial X/\partial L}{\partial X/\partial K} = \frac{b_1\left(\frac{X}{L}\right)}{b_2\left(\frac{X}{K}\right)}$$

$$= \frac{b_1}{b_2}\left(\frac{K}{L}\right)$$

CES prodn fn

$$Q = A\left[\delta K^{-\rho} + (1-\delta) L^{-\rho}\right]^{-\frac{1}{\rho}}$$

$A > 0$, $0 < \delta < 1$, $-1 < \rho \neq 0$

3) Elast of Subst

$$a = \frac{\partial\left(\frac{K}{L}\right) / \frac{K}{L}}{\partial(MRTS) / MRTS} = 1$$

$$a = \frac{\partial\left(\frac{K}{L}\right) / \frac{K}{L}}{d\left(\frac{b_1}{b_2} \cdot \frac{K}{L}\right) \Big/ \left(\frac{b_1}{b_2}\right)\left(\frac{K}{L}\right)} = \frac{d\left(\frac{K}{L}\right) / \frac{K}{L}}{d\left(\frac{K}{L}\right)\frac{b_1}{b_2} \Big/ \left(\frac{b_1}{b_2}\right)\left(\frac{K}{L}\right)}$$

$$= \frac{d\left(\frac{K}{L}\right)\left(\frac{b_1}{b_2}\right)}{\left(\frac{b_1}{b_2}\right) d\left(\frac{K}{L}\right)} = 1.$$

4) Factor intensity

In C.D. fn, it is measured by $\frac{b_1}{b_2}$. The higher the ratio, the more lab int the tech.

Lower the $\frac{b_1}{b_2}$, more cap int the tech

A → eff. parameter
δ → distⁿ parameter
ρ → substⁿ ″ ″

| homo of deg one

$A[\delta(jK)^{-\rho}+(1-\delta)(jL)^{-\rho}]^{-\frac{1}{\rho}}$

$= A\{j^{-\rho}[\delta K^{-\rho}+(1-\delta)L^{-\rho}]\}^{-\frac{1}{\rho}}$

$=(j^{-\rho})^{-\frac{1}{\rho}}Q = jQ$ → CRS

5) The eff is measured by b_0.
More eff. firm will have larger b_0.

6) RTS is measured by b_1+b_2

$MP_L = \frac{\partial x}{\partial L}$ $MP_K = \frac{\partial x}{\partial K}$

MPs of a factor may assume any value, positive, zero or negative. However the basic prodⁿ th concentrates only on the eff part of the prod fⁿ, that is on the range of output over which MPs are > 0. The th. of prodⁿ concentrates on the range of output over which MPs are > 0 but diminish ie over the range of diminishing MP of the factors of prodⁿ ∴ the ranges of output are A'B' and C'D'.

(*a*) Maximise profit Π, subject to a cost constraint. In this case total cost and prices are given ($\bar{C}, \bar{w}, \bar{r}, \bar{P}_x$), and the problem may be stated as follows

$$\max \Pi = R - \bar{C}$$
$$\Pi = \bar{P}_x X - \bar{C}$$

Clearly maximisation of Π is achieved in this case if X is maximised, since $\bar{C}$ and $\bar{P}_x$ are given constants by assumption.

(*b*) Maximise profit Π, for a given level of output. For example, a contractor wants to build a bridge (X is given) with the maximum profit. In this case we have

$$\max \Pi = R - C$$
$$\Pi = \bar{P}_x \bar{X} - C$$

Clearly maximisation of Π is achieved in this case if cost C is minimised, given that X and P_x are given constants by assumption.

The analysis will be carried out first by using diagrams and subsequently by applying calculus.

For a graphical presentation of the equilibrium of the firm (its profit-maximising position) we will use the isoquant map (figure 3.32) and the isocost-line(s) (figure 3.33).

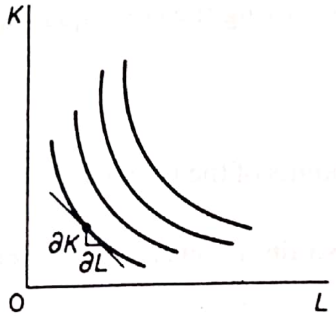

Figure 3.32

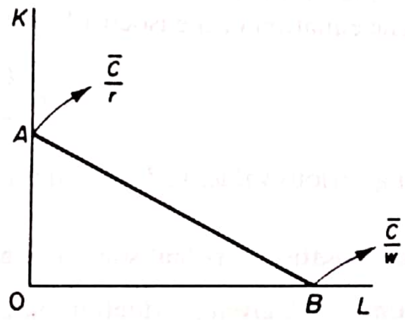

Figure 3.33

The isoquants have been explained in section I, where it was shown that the slope of an isoquant is

$$-\frac{\partial K}{\partial L} = MRS_{L,K} = \frac{MP_L}{MP_K} = \frac{\partial X/\partial L}{\partial X/\partial K}$$

The *isocost line* is defined by the cost equation

$$C = (r)(K) + (w)(L)$$

where w = wage rate, and r = price of capital services.

The isocost line is the locus of all combinations of factors the firm can purchase with a given monetary cost outlay.[1]

The slope of the isocost line is equal to the ratio of the prices of the factors of production:

$$\text{slope of isocost line} = \frac{w}{r}$$

[1] There is a close analogy between the consumer's budget line (Chapter 2, figure 2.10) and the firm's isocost line.

Proof

Assume that the total cost outlay the firm undertakes is $\bar{C}$. If the entrepreneur spends all the amount $\bar{C}$ on capital equipment, the maximum amount he can buy from this factor is

$$0A = \frac{\bar{C}}{r}$$

If all cost outlay is spent on labour the maximum amount of this factor that the firm can purchase is

$$0B = \frac{\bar{C}}{w}$$

The slope of the isocost line is

$$\frac{0A}{0B} = \frac{\bar{C}/r}{\bar{C}/w} = \frac{w}{r}$$

It can be shown that any point on the line AB satisfies the cost equation ($C = r \cdot K + w \cdot L$), so that, for given prices of the factors and for given expenditure on them, the isocost line shows the alternative combinations of K and L that can be purchased by the firm. The equation of the isocost line is found by solving the cost equation for K:

$$K = \frac{\bar{C}}{r} - \frac{w}{r}L$$

By assigning various values to L we can find all the points of the isocost line.

Case 1: maximisation of output subject to a cost constraint (financial constraint)

We assume: (*a*) A given production function

$$X = f(L, K, v, \gamma)$$

and (*b*) given factor prices, w, r, for labour and capital respectively.

The firm is in equilibrium when it maximizes its output given its total cost outlay and the prices of the factors, w and r.

In figure 3.34 we see that the maximum level of output the firm can produce, given the cost constraint, is X_2 defined by the tangency of the isocost line, and the highest isoquant. The optimal combination of factors of production is K_2 and L_2, for prices w and r. Higher levels of output (to the right of e) are desirable but not attainable due

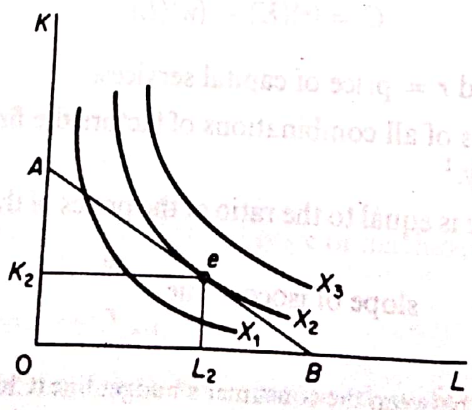

Figure 3.34

to the cost constraint. Other points on AB or below it lie on a lower isoquant than X_2. Hence X_2 is the maximum output possible under the above assumptions (of given cost outlay, given production function, and given factor prices). At the point of tangency (e) the slope of the isocost line (w/r) is equal to the slope of the isoquant (MP_L/MP_K). This constitutes the first condition for equilibrium. The second condition is that the isoquants be convex to the origin. In summary: the conditions for equilibrium of the firm are:

(*a*) Slope of isoquant = Slope of isocost

or
$$\frac{w}{r} = \frac{MP_L}{MP_K} = \frac{\partial X/\partial L}{\partial X/\partial K} = MRS_{L,K}$$

(*b*) The isoquants must be convex to the origin. If the isoquant is concave the point of tangency of the isocost and the isoquant curves does not define an equilibrium position (figure 3.35).

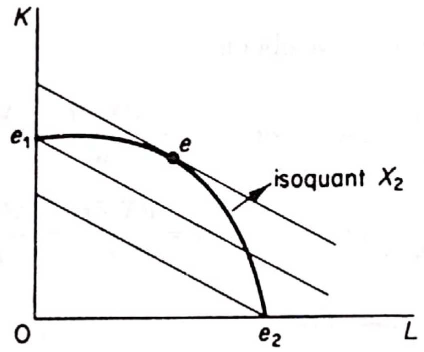

Figure 3.35

Output X_2 (depicted by the concave isoquant) can be produced with lower cost at e_2 which lies on a lower isocost curve than e. (With a concave isoquant we have a 'corne solution.)

Formal derivation of the equilibrium conditions

The equilibrium conditions may be obtained by applying calculus and solving a 'constrain maximum' problem which may be stated as follows. The rational entrepreneur seeks the ma misation of his output, given his total-cost outlay and the prices of factors. Formally:

Maximise $X = f(L, K)$

subject to $\bar{C} = wL + rK$ (cost constraint)

This is a problem of constrained maximum and the above conditions for the equilibrium of t firm may be obtained from its solution.

We can solve this problem by using Lagrangian multipliers. The solution involves the followi steps:

Rewrite the constraint in the form

$$\bar{C} - wL - rK = 0$$

Multiply the constraint by a constant λ which is the Lagrangian multiplier:

$$\lambda(\bar{C} - wL - rK) = 0$$

The Lagrangian multipliers are undefined constants which are used for solving constrain maxima or minima. Their value is determined simultaneously with the values of the oth unknowns (L and K in our example). There will be as many Lagrangian multipliers as there are constraints in the problem.

Form the 'composite' function

$$\phi = X + \lambda(\bar{C} - wL - rK)$$

It can be shown that maximisation of the ϕ function implies maximisation of the output.

The first condition for the maximisation of a function is that its partial derivatives be equal to zero. The partial derivatives of the above function with respect to L, K and λ are:

$$\frac{\partial \phi}{\partial L} = \frac{\partial X}{\partial L} + \lambda(-w) = 0 \tag{3.1}$$

$$\frac{\partial \phi}{\partial K} = \frac{\partial X}{\partial K} + \lambda(-r) = 0 \tag{3.2}$$

$$\frac{\partial \phi}{\partial \lambda} = \bar{C} - wL - rK = 0 \tag{3.3}$$

Solving the first two equations for λ we obtain

$$\frac{\partial X}{\partial L} = \lambda w \quad \text{or} \quad \lambda = \frac{\partial X/\partial L}{w} = \frac{MP_L}{w}$$

$$\frac{\partial X}{\partial K} = \lambda r \quad \text{or} \quad \lambda = \frac{\partial X/\partial K}{r} = \frac{MP_K}{r}$$

The two expressions must be equal; thus

$$\frac{\partial X/\partial L}{w} = \frac{\partial X/\partial K}{r} \quad \text{or} \quad \frac{MP_L}{MP_K} = \frac{\partial X/\partial L}{\partial X/\partial K} = \frac{w}{r}$$

This firm is in equilibrium when it equates the ratio of the marginal productivities of factors to the ratio of their prices.

It can be shown[1] that the second-order conditions for equilibrium of the firm require that the marginal product curves of the two factors have a negative slope.

The slope of the marginal product curve of labour is the second derivative of the production function:

$$\text{slope of } MP_L \text{ curve} = \frac{\partial^2 X}{\partial L^2}$$

Similarly for capital:

$$\text{slope of } MP_K \text{ curve} = \frac{\partial^2 X}{\partial K^2}$$

The second-order conditions are

$$\frac{\partial^2 X}{\partial L^2} < 0 \quad \text{and} \quad \frac{\partial^2 X}{\partial K^2} < 0$$

and

$$\left(\frac{\partial^2 X}{\partial L^2}\right)\left(\frac{\partial^2 X}{\partial K^2}\right) > \left(\frac{\partial^2 X}{\partial L\,\partial K}\right)^2$$

These conditions are sufficient for establishing the convexity of the isoquants.

[1] See Henderson and Quandt, *Microeconomic Theory* (McGraw-Hill, 1958) pp. 49–54.

Case 2: minimisation of cost for a given level of output

The conditions for equilibrium of the firm are formally the same as in Case 1. That is, there must be tangency of the (given) isoquant and the lowest possible isocost curve, and the isoquant must be convex. However, the problem is conceptually different in the case of cost minimisation. The entrepreneur wants to produce a given output (for example, a bridge, a building, or $\bar{X}$ tons of a commodity) with the minimum cost outlay.

In this case we have a single isoquant (figure 3.36) which denotes the desired level of output, but we have a set of isocost curves (figure 3.37). Curves closer to the origin show a lower total-cost outlay. The isocost lines are parallel because they are drawn on the assumption of constant prices of factors: since w and r do not change, all the isocost curves have the same slope w/r.

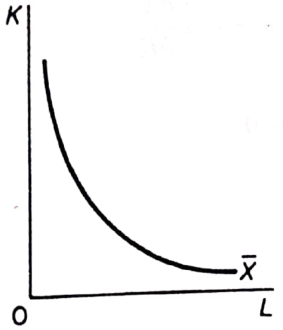

Figure 3.36

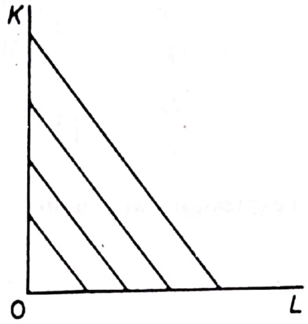

Figure 3.37

The firm minimises its costs by employing the combination of K and L determined by the point of tangency of the $\bar{X}$ isoquant with the lowest isocost line (figure 3.38). Points below e are desirable because they show lower cost but are not attainable for output $\bar{X}$. Points above e show higher costs. Hence point e is the least-cost point, the point denoting the least-cost combination of the factors K and L for producing $\bar{X}$.

Clearly the conditions for equilibrium (least cost) are the same as in Case 1, that is, equality of the slopes of the isoquant and the isocost curves, and convexity of the isoquant.

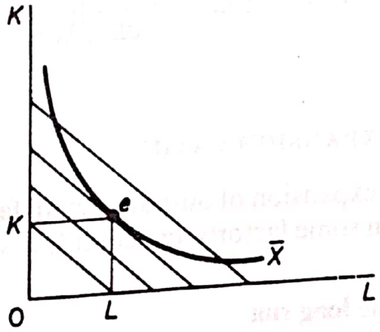

Figure 3.38

Formally:

Minimise

$$C = f(X) = wL + rK$$

subject to

$$\bar{X} = f(L, K)$$

Rewrite the constraint in the form

$$\bar{X} - f(L, K) = 0$$

Premultiply the constraint by the Lagrangian multiplier λ

$$\lambda(\bar{X} - f(L, K)) = 0$$

Form the 'composite' function

$$\phi = C - \lambda[\bar{X} - f(L, K)]$$

or

$$\phi = (wL + rK) - \lambda[\bar{X} - f(L, K)]$$

Take the partial derivatives of ϕ with respect to L, K and λ and equate to zero:

$$\frac{\partial \phi}{\partial L} = w - \lambda \frac{\partial f(L, K)}{\partial L} = 0 = w - \lambda \frac{\partial X}{\partial L}$$

$$\frac{\partial \phi}{K} = r - \lambda \frac{\partial f(L, K)}{\partial K} = 0 = r - \lambda \frac{\partial X}{\partial K}$$

$$\frac{\partial \phi}{\partial \lambda} = -[\bar{X} - f(L, K)] = 0$$

From the first two expressions we obtain

$$w = \lambda \frac{\partial X}{\partial L}$$

$$r = \lambda \frac{\partial X}{\partial K}$$

Dividing through these expressions we find

$$\frac{w}{r} = \frac{\partial X/\partial L}{\partial X/\partial K} = MRS_{L,K}$$

This condition is the same as in Case 1 above. The second (sufficient) condition, concerning the convexity of the isoquant, is fulfilled by the assumption of negative slopes of the marginal product of factors as in Case 1, that is

$$\frac{\partial^2 X}{\partial L^2} < 0, \quad \frac{\partial^2 X}{\partial K^2} < 0 \quad \text{and} \quad \left(\frac{\partial^2 X}{\partial L^2}\right)\left(\frac{\partial^2 X}{\partial K^2}\right) > \left(\frac{\partial^2 X}{\partial L\, \partial K}\right)^2$$

B. CHOICE OF OPTIMAL EXPANSION PATH

We distinguish two cases: expansion of output with all factors variable (the long run), and expansion of output with some factor(s) constant (the short run).

Optimal expansion path in the long run

In the long run all factors of production are variable. There is no limitation (technical or financial) to the expansion of output. The firm's objective is the choice of the optimal way of expanding its output, so as tc maximise its profits. With given factor prices (w, r) and given production function, the optimal expansion path is determined by the points of tangency of successive isocost lines and successive isoquants.

If the production function is homogeneous the expansion path will be a straight line through the origin, whose slope (which determines the optimal K/L ratio) depends on the ratio of the factor prices. In figure 3.39 the optimal expansion path will be $0A$,

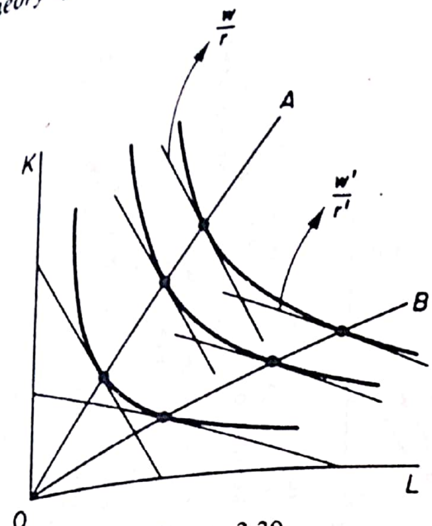

Figure 3.39

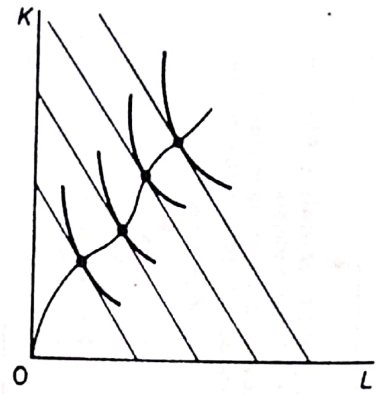

Figure 3.40

defined by the locus of points of tangency of the isoquants with successive parallel isocost lines with a slope of w/r. If the ratio of the prices increases the isocost lines become flatter (for example, with a slope of w'/r'), and the optimal expansion path will be the straight line $0B$. Of course, if the ratio of prices of factors was initially w/r and subsequently changes to w'/r', the expansion path changes: initially the firm moves along $0A$, but after the change in the factor prices it moves along $0B$.

If the production function is non-homogeneous the optimal expansion path will not be a straight line, even if the ratio of prices of factors remains constant. This is shown in figure 3.40. It is due to the fact that in equilibrium we must equate the (constant) w/r ratio with the $MRS_{L,K}$, which is the same on a curved isocline (see section II).

Optimal expansion path in the short run

In the short run, capital is constant and the firm is coerced to expand along a straight line parallel to the axis on which we measure the variable factor L. With the prices of factors constant the firm does not maximise its profits in the short run, due to the constraint of the given capital. This situation is shown in figure 3.41. The optimal expansion path would be $0A$ were it possible to increase K. Given the capital equipment, the firm can expand only along $\bar{K}\bar{K}$ in the short run.

The above discussion of the choice of optimal combination of the factors of production is schematically summarised on p. 94.

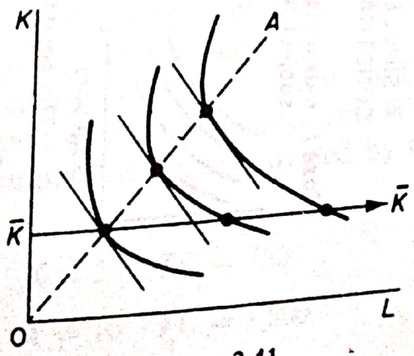

Figure 3.41

Plotting these points on a two-dimensional diagram with TC on the vertical axis and output (X) on the horizontal axis, we obtain the total-cost curve (figure 3.43). With our assumption (of constant returns to scale and of constant factor prices) the AC is constant (£1.50 per 'unit' of output), hence the AC will be a straight line, parallel to the horizontal axis (figure 3.44). It is important to remember that the cost curves assume that the problem of choice of the optimal (least-cost) technique has been solved at a previous stage. In other words, the complex problem of finding the cheapest combination of factor inputs must be solved before the cost curve is defined.

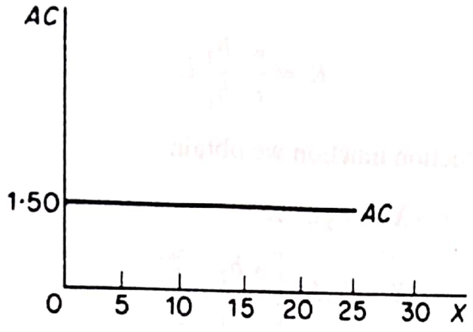

Figure 3.44

B. FORMAL DERIVATION OF COST CURVES FROM A PRODUCTION FUNCTION

In applied research one of the most commonly used forms of production function is the Cobb–Douglas form

$$X = b_0 L^{b_1} K^{b_2}$$

Given this production function and the cost equation

$$C = wL + rK$$

we want to derive the cost function, that is, the total cost as a function of output

$$C = f(X)$$

We begin by solving the constrained output maximisation problem:

Maximise $\quad X = b_0 L^{b_1} K^{b_2}$

subject to $\quad \bar{C} = wL + rK \quad$ (cost constraint)

(The bar on top of C has the meaning that the firm has a *given* amount of money to spend on both factors of production.)

We form the 'composite' function

$$\phi = X + \lambda(\bar{C} - wL - rK)$$

where λ = Lagrangian multiplier

The first condition for maximisation is that the first derivatives of the function with respect to L, K and λ be equal to zero:

$$\frac{\partial \phi}{\partial L} = b_1 \frac{X}{L} - \lambda w = 0 \qquad (3.4)$$

$$\frac{\partial \phi}{\partial K} = b_2 \frac{X}{K} - \lambda r = 0 \qquad (3.5)$$

$$\frac{\partial \phi}{\partial \lambda} = (C - wL - rK) = 0 \qquad (3.6)$$

From equations 3.4 and 3.5 we obtain

$$b_1 \frac{X}{L} = \lambda w \qquad \text{and} \qquad b_2 \frac{X}{K} = \lambda r$$

Dividing these expressions we obtain

$$\frac{b_1}{b_2} \cdot \frac{K}{L} = \frac{w}{r}$$

Solving for K

$$K = \frac{w}{r} \cdot \frac{b_2}{b_1} L \tag{3.7}$$

Substituting K into the production function we obtain

$$X = b_0 L^{b_1} K^{b_2}$$

$$X = b_0 L^{b_1} \left[\frac{w}{r} \frac{b_2}{b_1} L \right]^{b_2}$$

$$X = b_0 \left[\left(\frac{w}{r} \right) \left(\frac{b_2}{b_1} \right) \right]^{b_2} L^{(b_1 + b_2)}$$

The term in brackets is the constant term of the function: it includes the three coefficients of the production function, b_0, b_1, b_2, and the prices of the factors of production.

Solving the above form of the production function for L, we find

$$\frac{1}{b_0 \left(\frac{w}{r} \frac{b_2}{b_1} \right)^{b_2}} X = L^{(b_1 + b_2)}$$

or

$$\left[\frac{1}{b_0 \left(\frac{w}{r} \frac{b_2}{b_1} \right)^{b_2}} \cdot X \right]^{1/(b_1 + b_2)} = L$$

or

$$L = \left(\frac{rb_1}{wb_2} \right)^{b_2/(b_1 + b_2)} \left(\frac{X}{b_0} \right)^{1/(b_1 + b_2)} \tag{3.8}$$

Substituting the value of L from expression 3.8 into expression 3.7 for capital we obtain

$$K = \frac{w}{r} \cdot \frac{b_2}{b_1} L$$

$$K = \left(\frac{wb_2}{rb_1} \right)^{1/(b_1 + b_2)} \left(\frac{X}{b_0} \right)^{1/(b_1 + b_2)} \tag{3.9}$$

Substituting expression 3.8 and 3.9 into the cost equation $C = wL + rK$ we find

$$C = \left(\frac{1}{b_0} \right)^{1/(b_1 + b_2)} \left[w \left(\frac{rb_1}{wb_2} \right)^{b_2/(b_1 + b_2)} + r \left(\frac{wb_2}{rb_1} \right)^{b_1/(b_1 + b_2)} \right] \cdot X^{1/(b_1 + b_2)}$$

Some Mathematical Application on Cost of Production.

1. Let a firm has $SRTC = q^3 - 10q^2 + 17q + 66$. If $P = 5$, find q at which it maximises profit. Also calculate the output elasticity of cost at this output.

Ans: We know that $\pi = TR - TC$

$$= 5q - (q^3 - 10q^2 + 17q + 66)$$

Now for F.O.C. of π maximisation

$$\frac{\partial \pi}{\partial q} = 0$$

$$\Rightarrow 3q^2 - 20q + 12 = 0$$

$$\therefore q = 6, \frac{2}{3}, \quad \text{At } q = 6, \frac{\partial^2 \pi}{\partial q^2} = -16 < 0.$$

$\therefore$ Profit maximising output is 6.

Output elasticity of cost $= \frac{\partial q}{\partial c} \cdot \frac{c}{q}$

$$= \frac{AC}{MC}.$$

At $q = 6$, $AC = q^2 - 10q + 17 + \frac{66}{q}$

$$= 4.$$

$\therefore$ Output elasticity $= \frac{4}{5} = 0.8$.

2. Let production fun^n is $Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$ and cost is $C = rK + WL$. Find out cost function.

The equation of expansion path is.

$$\frac{MP_L}{MP_K} = \frac{W}{r}.$$

$$\Rightarrow K = \frac{W}{r} L \quad \& \quad L = \frac{r}{W} K.$$

$$\therefore Q = \left(\frac{W}{r} L\right)^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$\therefore L = \sqrt{\frac{r}{W}} \cdot Q, \quad K = \sqrt{\frac{W}{r}} \cdot Q.$$

$$\therefore C = r\sqrt{\frac{W}{r}}\, Q + W \cdot \sqrt{\frac{r}{W}}\, Q.$$

$$= 2\sqrt{Wr}\, Q.$$

This is the simplified version of cost function which is straight line through the origin with slope $2\sqrt{Wr}$.

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CC 202 ME: Managerial Economics

Module –II

Unit 5 : Risk and Uncertainties in Managerial Decision Making

(Dr. Samarpita Seth)

Paper CC 202: Managerial Economics

Module II

Topic 5: Risk and Uncertainties in Managerial Decision Making

(*) Concepts of lottery, definitions and features of Expected Utility Function (also called VNM Utility functions): are discussed in the class.

Ref: Microeconomic Analysis
Hal. R. Varian.

(*) Preference Towards Risk.

Ref: Microeconomics
Pindyck, Rubinfeld and Mehta

People differ in their willingness to bear risk. Some are risk averse, some are risk lover and some are risk neutral. To understand difference in risk bearing we start with an example.

Suppose a consumer has an income Rs 15,000 and she is considering a new but risky job with two alternative

(1) Rs. 30,000 with probability 0.5

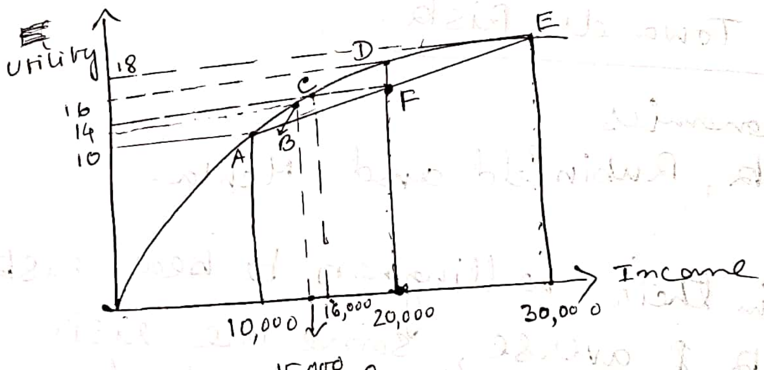
(2) Rs 10,000 with probability 0.5.

Naturally we can calculate expected Income of the future job as follows

$0.5(30{,}000) + 0.5(10{,}000) = \text{Rs } 20{,}000$.

Clearly as Rs 20,000 > Rs 15,000, the consumer will accept the risky job by leaving her present certain income if she is risk lover. If she is risk averse then she will prefer to stay on current job. And she will be indifferent between present and future job if she is risk neutral.

The same conclusion can by interpreted with the help of following diagram.



Risk Averse

fig 1.

Fig 1. explained the above example.

~~Suppose~~ We assume $U(\text{income } 10{,}000) = 10$ (point A).

$U(\text{'' } 30{,}000) = 18$ (point E)

Hence $\frac{1}{2} \cdot 10 + \frac{1}{2} 18 = 14$ (point F)

~~Hence~~. Clearly, point B corresponds to the utility of certain present income Rs 15,000.

Lastly point D corresponds to the utility of certain income Rs 20,000.

~~∴ EU(certain income)~~
Since utility of certain income is greater than utility of uncertain income (exp income) in figure 1
[Rs 20,000 = 0.5 (10,000) + 0.5 (30,000)]

consumer is risk averse.

Similarly when consumer is risk lover her utility function will be convex like fig 2

Risk lover.

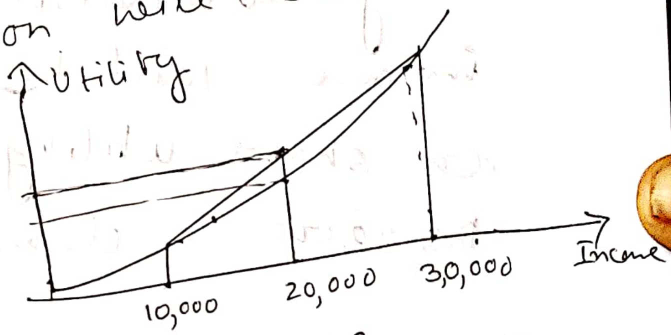
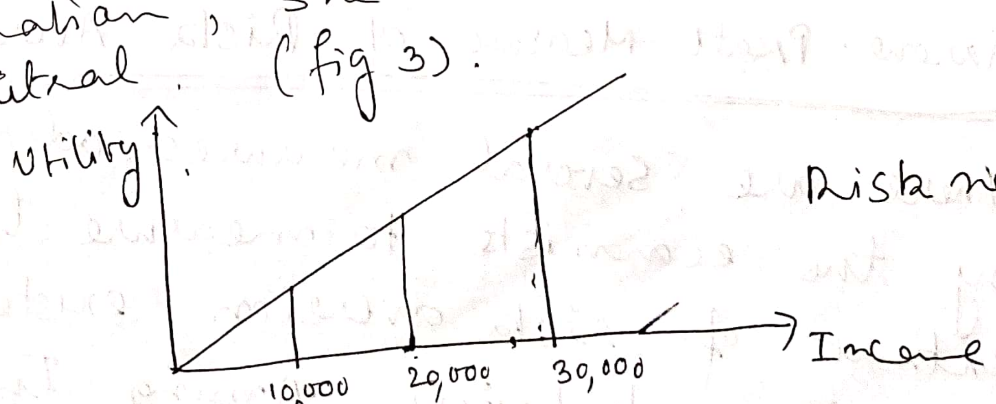


fig 2.

Clearly, utility of certain income of Rs 20,000 is less than utility of exp. income Rs 20,000. So consumer prefers risk.

Lastly when utility of certain income is same as utility of expected income consumer is indifferent between two situation, she is said to be risk neutral (fig 3).



Risk neutral

fig 3

Clearly utility function is linear.

Risk Premium: Risk Premium is the maximum amount of money a risk-averse person will pay to avoid taking a risk. The magnitude of risk premium is shown by $\overline{CF}$ in fig 1.

We already have shown that with the risky alternative consumer's expected income will be Rs 20,000. point F shows utility corresponding to Rs 20,000. clearly U(F) = 14.

Now, consumer can attain the same amount of utility (=14) with certain income Rs 16,000. This is shown by point C in figure 1. Hence

$\overline{CF}$ = 20,000 − 16,000 = Rs 4,000 is the risk premium in the present risk averse consumer's point of view.

Arrow. Pratt Measure of Risk Aversion

measure was proposed by economists Arrow and Pratt.

Arrow-Pratt measure is given by the formula.

$$A = -\frac{u''(x)}{u'(x)}.$$

where x is the income of the consumer $u(x)$ is the ~~util~~ utility corresponding to x.

Clearly $A = -\frac{u''(\cdot)}{u'(\cdot)} > 0$.

if $u'' < 0$, $u' > 0$

$\Rightarrow$ utility function is concave (fig 1).

$$A = -\frac{u''(\cdot)}{u'(\cdot)} < 0$$

if $u'' > 0$, $u' > 0$

$\Rightarrow$ utility function is convex (fig 2)

$$A = -\frac{u''(\cdot)}{u'(\cdot)} = 0$$

if $u'' = 0$, $u' > 0$

$\Rightarrow$ utility function is linear. (fig 3).

Risk Aversion and Indifference Curve.

To understand the relationship between risk aversion and indifference curve. we start with an example. Suppose one person, seeking job in the market, found following two alternative jobs.

Job 1:
- Income Rs 2000 (prob 0.5)
- Income Rs 1000 (prob 0.5)

Job 2:
- Income Rs 1510 (prob 0.99)
- Income Rs 510 (prob 0.01)

Expected income for job 1:

$0.5\,(Rs\,2000) + 0.5\,(Rs\,1000) = Rs\ 1500$ --- (1)

Expected Income for job 2

$0.99\,(Rs\,1510) + 0.01\,(Rs\,510) = Rs\ 1500$ --- (2)

So both job will give same expected income / mean income for the job seeker. However, variability of two jobs will be different. This variability /riskiness of these jobs are determined by standard deviation (root mean-square deviation).

Std. deviation of job 1 is given by:

$\sqrt{0.5\,(2000-1500)^2 + 0.5\,(1000-1500)^2}$

$= \sqrt{0.5\,(250{,}000) + 0.5\,(250{,}000)}$

$= \sqrt{250{,}000} = \text{Rs } 500$ --- (3)

Similarly standard deviation of job2 is given by

$\sqrt{0.99\,(1510-1500)^2 + 0.01\,(510-1500)^2}$

$= \sqrt{0.99\,(100) + 0.01\,(980{,}100)}$

$= \sqrt{9900} = \text{Rs } 99.50$ --- (4)

From Expressions (1), (2), (3), (4) we can conclude

(I) both job will earn same expected (mean) return for job seeker.

(II) std deviation of job1 is higher while std. deviation of job 2 is lower indicating that second job is less risky than first job.

So if jobseeker is a riskaverse person he/she will choose job2 and job1 will be chosen by a risk lover.

Applying the concept of mean return and variability/ riskiness of return. the relationship of Risk Aversion and Indifference curve/ utility levels can be stated in the following manner:

$U = E(R) - \frac{1}{2} A * \text{variance}$

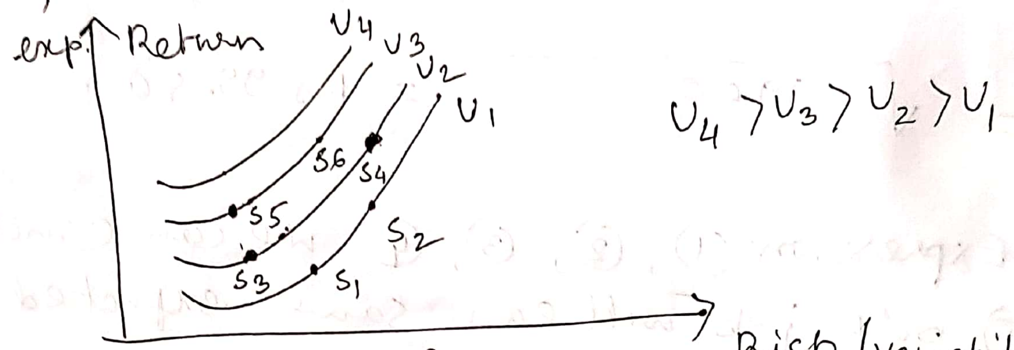
Where E(R) ⇒ expected/mean return

A ⇒ level of risk averseness

variance ⇒ $(S.D.)^2$ showing variability or riskiness of return.

U is the utility level obtained for any return.

To find out indifference curve utility level should remain constant for any risk, return combination



$U_4 > U_3 > U_2 > U_1$

fig 4

Fig 4 shows that (S_1, S_2) will give same level of utility. Similarly for (S_3, S_4) or (S_5, S_6).

* Comparing S_3 with S_5 ⇒ Risk/variability same at S_3 and S_5. But exp. return at S_5 > exp. return at S_3. So consumer will obtain higher utility at S_5 compared to at S_3.

* Comparing S_4 with S_6: Return same

in two points But variability is higher at S_4 than at S_6.

So we can conclude that utility level is positively related to mean return and negatively related to variability of the return.

Now we consider the relationship between level of risk aversion and utility with the help of following ~~dig~~ diagrams.

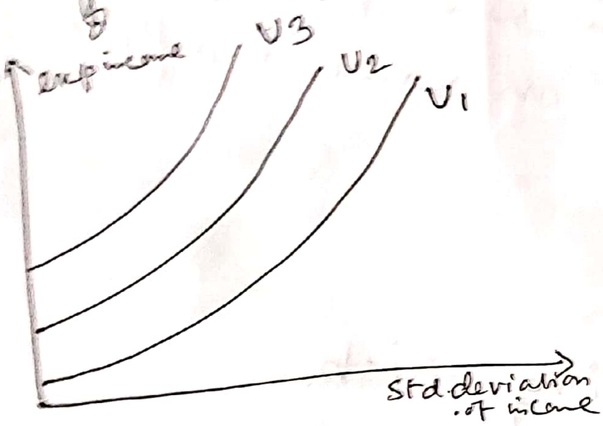


fig 5

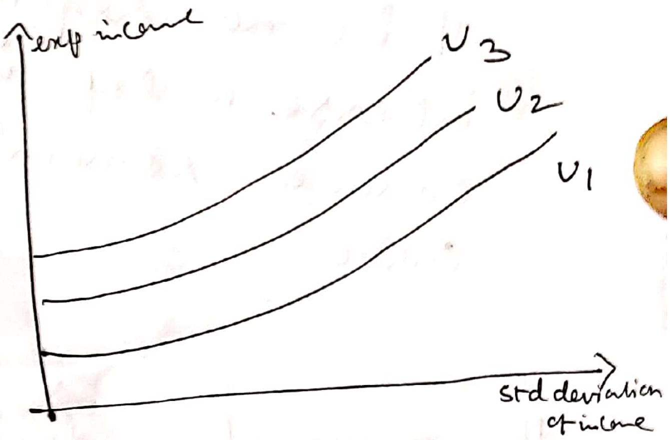


fig 6.

fig 6 shows indifference curve of any consumer who is slightly risk averse while in fig 5 consumer is highly risk averse.

Indifference curves are upward sloping because as variability / risk is negatively related to utility, higher risk should be accompanied by higher exp. return / income to keep utility level fixed / Constant.

⇒ From fig 6, it is clear that as consumer is slightly risk averse (risk aversion level is low), ~~large~~ large increase in standard deviation of income requires only a small increase in exp. income to leave consumer remain on same indifference curve.

⇒ In ~~const~~ contrast to that fig 5 shows consumer with high level of risk aversion. So an increase in standard deviation of income require ~~only~~ a large increase in exp income to leave the consumer remain on same indifference curve.

The process of reducing risk

There are several ways by which consumers and investors can reduce risk of choice of their investments. We are describing some common measures to reduce risk:

1. Insurance
2. Diversification of portfolios
3. Obtaining more information about choices and payoff.

First of all any risk averse person will always try to avoid risk by purchasing insurance. They will be willing to pay more on the insurance that will ~~covers~~ help them to recover fully / maximum amount of financial loss consumer might suffer.

For the insurance companies, "law of large number" is followed as principle. This means insurance companies will try to sell off their product to a large number of consumers. to avoid paying full amount (equivalent to the premium) to the beneficiary. In this way, larger the number of consumer purchasing insurance policies, lesser will be risk involved for the insurance companies.

Secondly, another method of reduction of risk is diversification of portfolios. This means. investor will alway try to invest their money in as many activities as possible. instead of keeping the entire money in a single activity. Hence more the diversification, lesser is the risk.

Thirdly, consumer / investor will try to acquire more information about the market outcome. Because higher the information availability lesser will be the gap between expected income and actual income from any specific investment / choice of market outcome.

Sometimes to reduce risk, investor / consumer might be ready to pay for acquiring information from some reliable / reputed sources.

CC 202: Managerial Economics (ME)

Module –II

Unit 5 : Risk and Uncertainties in Managerial Decision Making

(Pallavi Julasaria)

25 marks → I M   18.4.2012

Topic : Choice and Decision under Uncertainty.

Uncertainty :- there is an outcome. & there is a probability attached with the outcome.

Certain event : when it really happens.

Optimisation plan of a Decision maker under uncertainty. → Topic of discussion

How is ↓ affected under uncertainty.

consider → 2 events → $x : p$ [x occurring with probability p]
then, $y : 1-p$ [∵ sum is 1]

Here, x, y → outcomes [certain]
$p, 1-p$ → probabilities.

$L = \{x : p ; y : 1-p\}$ Lottery.

If there is more than 2 outcomes, then lottery is
$L = \{x_1 : p_1 ; x_2 : p_2 , ---- x_n : p_n\}$ where $\sum_{i=1}^{n} p_i = 1$
for all $i = 1, ---, n$

We define this as simple Lottery.

Properties of a Simple Lottery :-

Let us take ex as :- $L = \{x : 1 ; y : 0\}$ → certain event
But since probabilities exists → not exactly x occurs with a certainty but almost equal to certainty.

PROPERTY 1. Indifference : $L = \{x : 1 ; y : 0\} \approx x$
(not absolutely certain, 99.99% chance, but there is prob $\approx x$. y doesn't exists) b/c it just approaching the point.
For all practical purposes, take as $\approx x$.

2. Reflexivity : $L_1 = \{x : p ; y : 1-p\} \approx L_2 \{y : 1-p ; x : p\}$

Definition of Compound Lottery :- consider a lottery $L = \{x : p ; y : 1-p\}$
another, $L' = \{x : q ; L = 1-q\}$ → one of outcome is lottery itself.

One outcome is certain, other is lottery.

$L = \{x : p ; y : 1-p\}$

$L' = \{x : q ; L : 1-q\}$

$x : q + p(1-q)$ [∵ x occurs if L occurs, & L occurs with prob 1-q]

$y : (1-p)(1-q)$

$L' = \{x : q + p(1-q) ; y : (1-p)(1-q)\}$

↳ Simple lottery.

if both events is a lottery :-

$L_1 = \{x : p ; y : 1-p\}$

$L_2 = \{x : q ; y = 1-q\}$

$L = \{L_1 : r ; L_2 : 1-r\}$

$x : rp + (1-r)q$

$y : r(1-p) + (1-r)(1-q)$

$L = \{x : rp + (1-r)q ; y : r(1-p) + (1-r)(1-q)\}$

↳ A simple lottery

PROPERTY 3 — All compound lotteries can be broken down as simple lotteries.

An advantage → we can show results in terms of simple lotteries.

Another Concept — RISK

Difference b/w Risk & Uncertainty

Risk : $L = \{x : p ; y : 1-p\}$

situation where, x, y known & p also known.

Pure Uncertainty :- x, y known, p unknown.

If we know rel^n b/w the probabilities, know some of properties of prob.

have got other info of probability.

Uncertainty : other info regarding p is there, not the values.

Both of them obeys identical rules, same
(Risk & Un) theorems & results).
(in some books, treated same)

Choice under uncertainty.

UTILITY : Choice
measure of lottery.

$L = \{x : p ; y : 1-p\}$

In case of lottery, we talk about Expected Utility :- [Additive way]

$$EU(L) = pU(x) + (1-p)U(y)$$

This f^n is famous as — Von Neumann – Morgenstern Utility f^n, worked in Risk context.

In the case of n out comes

$$EU(L) = \sum_{i=1}^{n} p_i U(x_i)$$

Jensen's Ineq.

$$U\left(\sum_{i=1}^{n} p_i x_i\right) \geq \sum_{i=1}^{n} p_i U(x_i).$$

$x : x_i$ —— x_n, $p : p_i$ —— p_n.

27.9.2012

Choice under Uncertainty :-

Outcome $x \rightarrow$ associated with probability 'p'
How ind maximises, maximises utility f^n. Ind doesn't know whether x will occur or not.

Outcome $y \rightarrow$ associated with prob $1-p$

∴ Individual faces a lottery.
Person optimises the Expected Utility :
Expected Utility : associated with lottery.

$$EU = pU(x) + (1-p)U(y)$$

Von Neumann – Morgenstern Utility function.

Diff b/w :-
Risk & Uncertainty.
↙ ↓
p is known   p unknown.

Calculations remains undisturbed (~~all~~ the Risk or Uncertainty)

Considering a Uncertain Situation :-

$x : p$ & $y : 1-p$

Risk Aversion :- avoid risk
certain outcome pref to risk.

Original U pref to Expec Utility.

$$EU = p\,U(x) + (1-p)\,U(y)$$

This is $\leq$ to :-

How to write Risk Aversion in Expression.

What is comparable & have to :

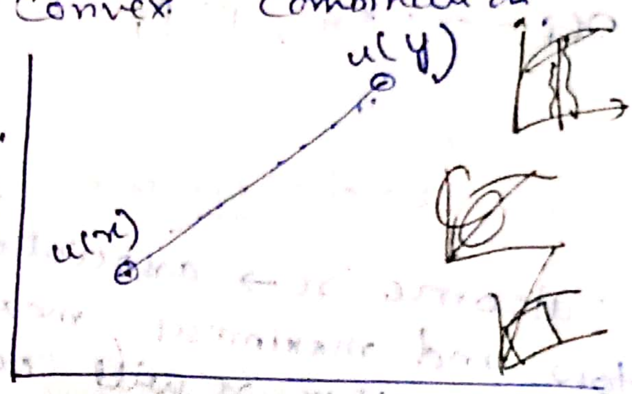
Certainty Equivalent to Lottery :-

If all expected U → convex

Exp U lies b/w u(x) & u(y).

$px + (1-p)y$

[convex combi b/w x & y]



Risk Aversion: expected utility of expected wealth ≥ expected utility

$$U\{px + (1-p)y\} \geq pU(x) + (1-p)U(y)$$

$kx + (1-k)y$

for all, $0 \leq k \leq 1$

Certainty equivalent of lottery — its utility exceeds U of Expected Utility.

Neutral :- = hold.

Risk lover :- convex — prefer uncertain situation to certainty.

$$U\{px + (1-p)y\} \leq pU(x) + (1-p)U(y)$$

Considering always :- n Variable system
bcz everything can't be worked out in 2 variable.

We will always have to deal with n variable.

Properties to Expected utility f^n :-

The linear f^n.

Whether expected utility f^n is convex to origin or not? E(U) follow same scale as U(x) & U(y). [H.W]

concave f^n.

Risk Aversion :- condition.

$U\{px + (1-p)y\} \geq pU(x) + (1-p)U(y)$

equality is borderline b/w Risk lover & aversion.

Objective Probability

Subjective Probability

All uncertain not objective. depends on value judgements. perception diff from person to person. p's may not be same for n & y is same. all the individuals.

2 contexts of using probability

[Subjective probability] :- Ex - "It is very probable that the democratic system of Govt India will adhere to till the end of this century". Probability here means the degree of belief in the proposition of the person making the statement.

[Objective Probability] :- Probability used in regard to the outcomes of an experiment that can conceivably be repeated an infinite no. of times under essentially similar conditions. The outcomes will be called events. Ex - Drawing a red ball from a urn of 20 balls. The probability here refers to the proportion of cases in which the event occurs in such repetitions of the experiment.

28-9-2012

Risk-Aversion :- particular wealth W_0. we are having

I have 2 options. have certain income to add to this wealth. (Z)

$$U(W_0 + Z)$$

$$L = \{x : p ; y : 1-p\}$$

Expected utility. $(W_0+x) : p$, $(W_0+y) : 1-p$ } $W_0 + L$

utility from certain Income ≥ Expected Utility of lottery.

$$U(W_0+Z) \geq EU\{W_0+L\}$$

Risk Aversion :- $U(W_0+Z) \geq pU(W_0+x) + (1-p)U(W_0+y)$

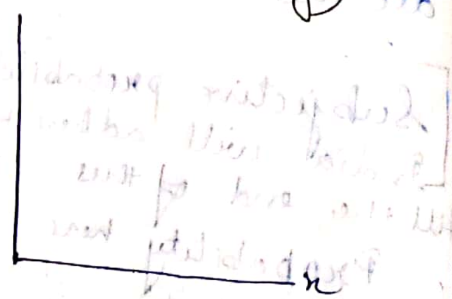
Equality is indifference. When equality holds, indiff b/w buying lottery or not.

Equality means Acceptance level of Risk.

$$U(W_0+Z) = pU(W_0+x) + (1-p)U(W_0+y)$$

$\{$ the x & y are variables.

Trying to look at shape of IC s so we differentiate the eq^n to look at slope.



In L.H.S ⇒ W_0+Z constant. $\frac{d}{dx}$.

$$0 = pU'(W_0+x) + (1-p)U'(W_0+y)\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = -\frac{pU'(W_0+x)}{(1-p)U'(W_0+y)}$$

$$= -\frac{p}{1-p}\frac{U'(W_0+x)}{U'(W_0+y)}$$

Slope of I.C.

$$\left.\frac{dy}{dx}\right|_{(0,0)} = -\frac{p}{1-p}\frac{U'(W_0)}{U'(W_0)}$$ (given probabilities it's const)

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Risk Aversion.

Jensen's Inequality

If there are 2 outcomes x_1, x_2 with probabilities p, $1-p$. then person is risk averse

$$U\{p x_1 + (1-p) x_2\} \geq pU(x_1) + (1-p)U(x_2)$$

Certain utility — Expected Utility.

person getting under certain condition is exceeding expected utility.

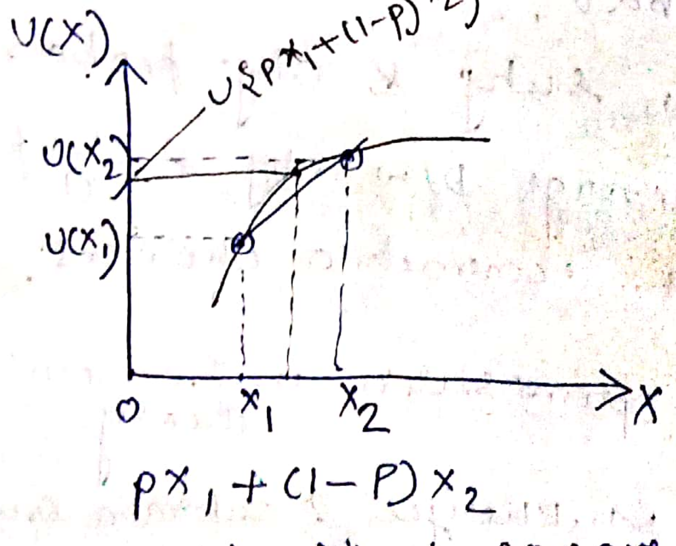
Lottery of n outcomes –

$$\{x_1; p_1 ; x_2 : p_2 ; \quad \text{---} \quad x_n : p_n\}$$

$$\sum_{i=1}^{n} p_i = 1.$$

$$U\left\{\sum_{i=1}^{n} p_i x_i\right\} \geq \sum_{i=1}^{n} p_i U(x_i)$$

Jehnsen's Inequality.



Geometry –

curve lying above the chord.

join $U(x_1)$ & $U(x_2)$.

LHS → points lying on utility f^n.

curve lying above chord.

If utility f^n is concave f^n, then the person is risk averse

$U'' < 0$

CHOICE UNDER UNCERTAINITY

Topic Name:-

Q1) What is meant by risk aversion? why are some people likely to be risk averse while others are risk lovers?

Show that concave utility of money function of an individual implies that she is risk averse.

Ans Risk aversion means preferring a certain income to a risky income with the same expected value. Risk aversion is the most common attitude toward risk.

A fair gamble is a gamble whose expected return is 0.

Individuals are said to be risk averse if they are not willing to undertake a fair gamble.

Risk neutral persons are indifferent between accepting or rejecting a fair gamble.

Risk loving individuals are eager to undertake a fair gamble.

Utility function of income will be different for these 3 groups of individuals. Expected utility theory helps us to differentiate between them.

We can explain this with the help of an example. Suppose a person have either a certain income of Rs 20,000 or a job yielding an income of Rs 30,000 with the probability 0.5(½) and an income of Rs 10,000 with probability 0.5 (½).

Thus expected income is $\frac{1}{2}(10,000) + \frac{1}{2}(30,000)$

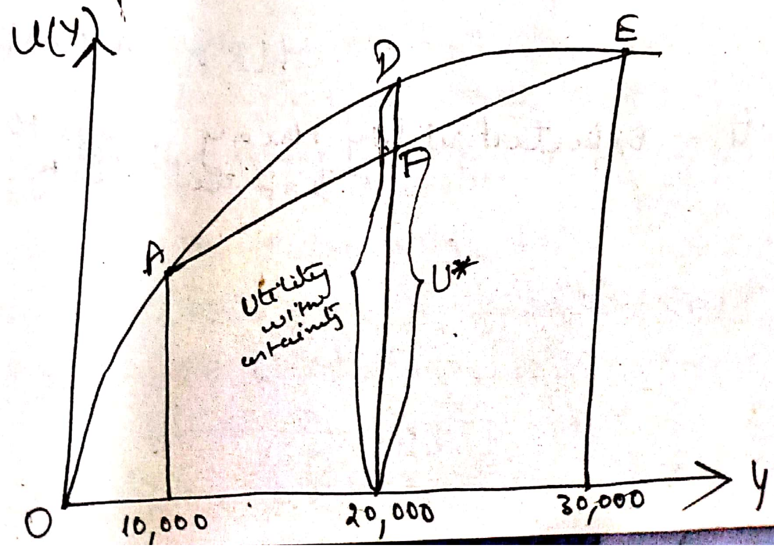
$= $ Rs 20,000.

For a risk averse person, marginal utility is diminishing, thus have a concave utility function.

With income 10,000, utility is given by point A — is 10, & utility is 18 at E with Rs 30,000. The expected utility of uncertain income is 14 — an average of utility at point A (10) & utility at E (18) — is shown by point F. Let this expected utility be U*.

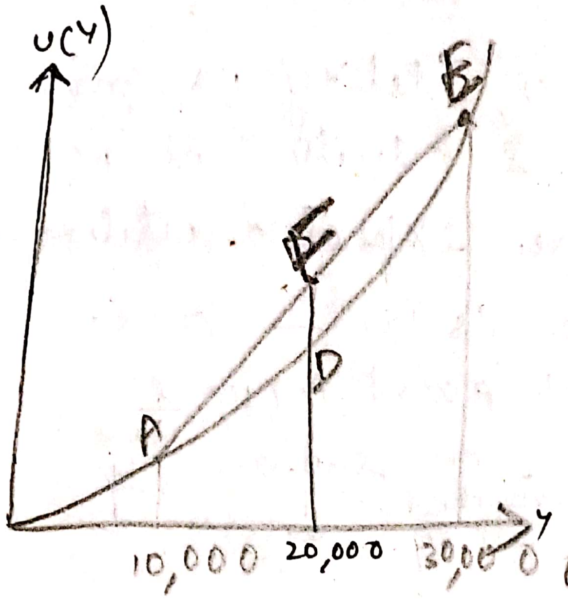
Utility generated if Rs 20,000 were earned without risk (with certainty) is 16 given at point D.

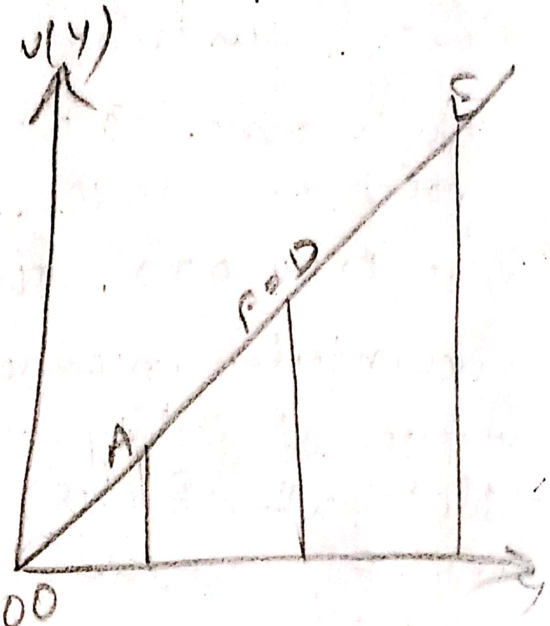
The expected utility associated with risky job (U*) 14 is less than utility 16 if Rs 20,000 is earned with certainty.



Thus, risk averse person will not take the gamble,

For risk averse person, losses are more important (in terms of change in utility) than gains. A 10,000 ↑se in income from Rs 20,000 to Rs 30,000 generates increase in utility of 2 units, a Rs 10,000 decrease in income from Rs 20,000 to Rs 10,000 creates a loss of utility of 6 units.





U* > U(20,000)

Risk Lover

prefers an uncertain income to certain income.

U* = U (20,000)

So utility with risk & utility with certain income is same

Risk Neutral.

So indifferent b/w certain & uncertain income.

MU of income is constant.

[Note - Expected utility theory says to maximise expected utility.]

Q) How can risk be reduced through diversification?

Ans Diversification means reducing risk ~~through~~ by ~~ac~~ allocating resources to a variety of activities whose outcomes are not closely related.

If we consider example of a job selling appliances – air conditioners and heaters. Of course, seller can not be sure how hot or cold the weather will be next year.

If There is 0.5 probability that it will be relatively hot year and 0.5 probability that it will be cold,

If the seller sells only ACs or only heaters, his actual income will be either Rs 12,000 or 30,000.

Income from sales of Appliances		
	Hot weather	Cold Weather
AC	30,000	12,000
Heater	12,000	30,000

His expected income will be → 0.5(30,000) + 0.5(12,000)
= 21,000

But, if seller diversify by dividing of time evenly between the two products.

In that case, his income will be certainly ~~Rs 12,000~~ Rs 21,000 regardless of weather.

If weather is hot, he will earn Rs 15,000 from Air conditions sales and Rs 6000 from heater sales; if it is cold, he will earn Rs 6000 from ACs and Rs 15,000 from heaters. Thus diversification eliminates risks of earning very low income of Rs12,000, in case weather is not favourable to the product the seller sells.

Thus Risk can be minimised by diversification — by allocating time so that he sell two or more products (whose sales are not closely related) rather than a single product.

Risk can be eliminated by diversification if sales are of negatively correlated variables.

Numerical sums example :-

Q) A risk averse person is offered a choice between a gamble paying him Rs 1000 with probability 0.25 and Rs 100 with probability 0.75 or a certain ~~inc~~ payment of Rs325. Which would he choose? Explain briefly.

Ans. The expected return from the gamble is

$$= 0.25(1000) + 0.75(100)$$
$$= 250 + 75$$
$$= 325$$

Another choice available to the risk averse person is of a certain income of Rs 325.

Thus the expected return from the gamble and the certain income is same.

But a risk averse person chooses a certain income to a risky income with same expected value.

(iii) Though probability of getting a very high income Rs 1000 is 0.25 but the probability of getting low income Rs 100 is much higher = 0.75.

Thus the risk could be measured by standard deviation is

$$= \sqrt{\frac{\sum p(x_i - \bar{x})^2}{\sum p}}$$ where p = probabilities, $\bar{x}$ = expected income, x_i = income.

$$= \sqrt{0.25(1000-325)^2 + 0.75(100-325)^2}$$

$$= \sqrt{225}$$

Thus there is a risk of 225 units.

But there is no such risk associated with certain payment of Rs 325.

Therefore the risk averse person will consider the loss more than the probable gain and would choose the certain income of Rs 325.

[SD - Square root of the weighted average of the squares of the deviations of the payoffs associated with each outcome from their expected return].

Ans 07) Expected income from the lottery will be = 0.1(100) + 0.2(50) + 0.7(10)

= 10 + 10 + 7

= 27

Q) What would a risk neutral person would pay to play the lottery?

Ans ~~A risk~~ expected value – Probability weighted average of the payoffs associated with all possible outcomes.

$$E(X) = Pr_1 X_1 + Pr_2 X_2 + \cdots + Pr_n X_n.$$

A risk neutral person is indifferent between a certain and an uncertain amount with same expected income.

~~To~~ Marginal utility from income is constant for a risk-neutral person thus a st line from ~~orig~~ origin.

Thus expected utility from the income from lottery is equal to the utility from certain income. As utilities do not differ, risk neutral would not be ready to pay ~~for~~ to play the lottery.

He would not be ready to pay for a risky income knowing that the riskfree income yields same utility.

Risk lover could pay to play the lottery but risk neutral could not.

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Q Derive the budget line of an investor facing a choice between a risky asset and risk-free asset using standard deviation of return as a measure of riskiness. Show that the equilibrium of a risk loving and a risk-averse investor using indifference curves under above situation.

Something that provides a flow of money or services to its owner is called asset.

An asset may be risky and riskless (or risk free) asset.

A risky asset that provides an uncertain flow of money or services to its owner.

Ex – corporate bonds where issuing corporation could go bankrupt and fail to pay bond owners their interest and principal. Long term Indian Government bonds that mature in 10-15 yrs could be risky because of inflation, the eventual payment could be worthless in real terms.

Risk-free assets are those assets that provides a flow of money or services that is known with certainty.

Ex – Short term Indian Government bonds – called Treasury bills are riskless as they mature in few months and Indian Government will probably not default the bond.

Expected Return is the return that

an asset earn on average.

Although risky assets like stocks have higher expected return than riskless asset, they carry much more risk.

Risk is measured by the standard deviation of real annual return.

Given that, the investor faces a choice between a risky and risk free asset, he will allocate his budget between these two assets and try to maximise utility.

Let the risk-free return on riskless asset be R_f, [Actual Return = expected return]

and expected return from investing in risky asset be R_m [actual return is r_m].

The risky asset will have a higher expected return than risk-free asset ($R_m > R_f$)

Investment portfolio — lets set b equal to fraction of investor's savings placed in risky asset and (1−b) used to purchase riskless asset.

∴ Expected return on total portfolio R_p is a weighted average of expected return on two assets =

$$R_p = bR_m + (1-b)R_f \quad \text{—(1)}$$

(risky) (risk-free)

Riskiness of the portfolio could be measured by standard deviation of portfolio, σ_p (with one risky and one risk-free asset) is the fraction of the portfolio invested in the risky asset times the standard deviation of that asset —

$$\sigma_p = b\,\sigma_m \qquad \text{—— (2)}$$

To determine the investor's decision to ~~accom~~ invest in risky a risk free asset i.e to determine fraction b, we must show that she faces a risk-return tradeoff analogous to consumer's budget line,

eqn (1) can be written as

$$R_p = R_f + b(R_m - R_f) \quad \text{—— (3)}$$

from (2), $b = \frac{\sigma_p}{\sigma_m}$

Putting in (3)

$$R_p = R_f + \frac{(R_m - R_f)}{\sigma_m}\sigma_p$$

This equation is a budget line because it describes the trade-off between risk (σ_p) and expected return (R_p)

Because R_m, R_f & σ_m are constants, the slope $\frac{R_m - R_f}{\sigma_m}$ is constant, as is the intercept R_f.

The equation says that the expected return of the portfolio increases as the standard deviation of that return (risk σ_p) increases.

The slope of the budget line $\frac{R_m - R_f}{\sigma_m}$ is called the price of risk because it tells us how much extra risk an investor must incur to enjoy a higher expected return.

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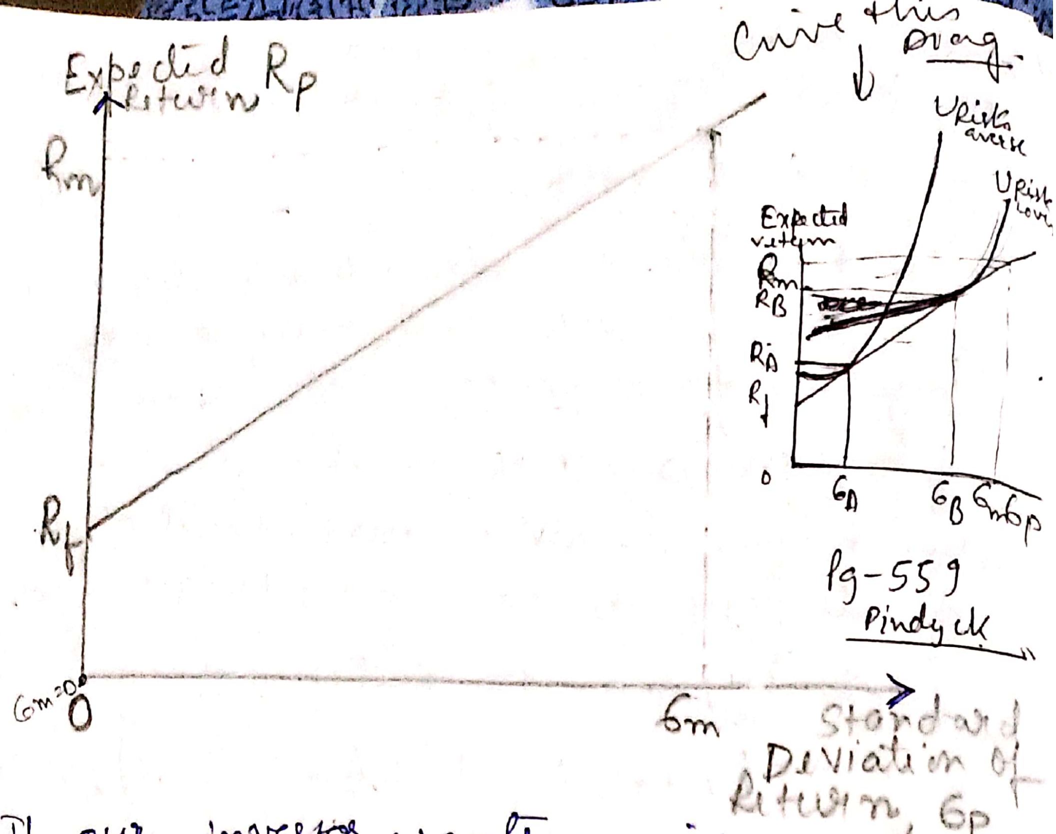


If our investor wants no risk she can invest all her funds in risk-free asset (b=0) and earn an expected return R_f. To receive a higher expected return she must incur some risk. For example, she could invest all her funds in stocks (b=1) earning an expected return R_m but incurring a standard deviation 6_m. Or she might invest some fraction of her funds in each type of asset, earning an expected return somewhere between R_f and R_m and facing a standard deviation less than 6_m but greater than 0.

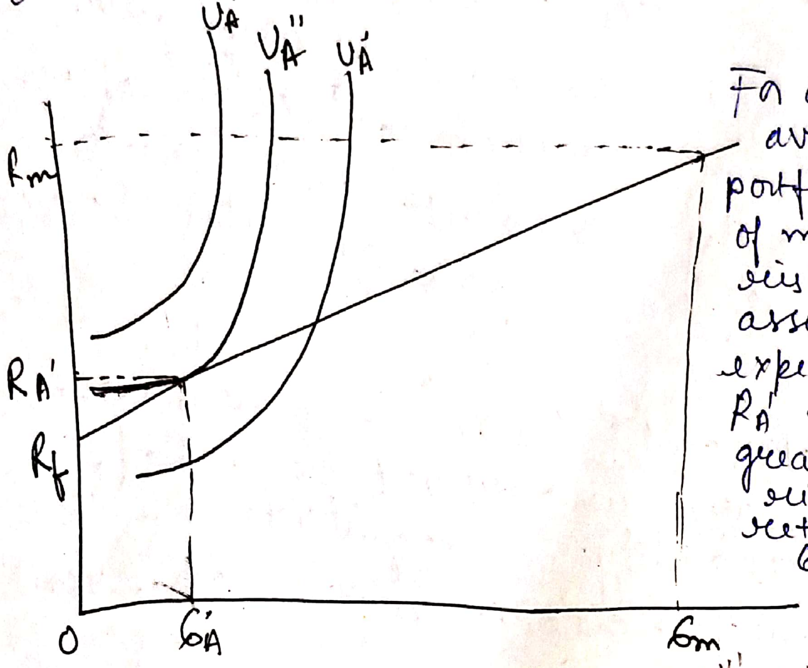
Equilibrium of Risk averse & Risk lover –
Indifference curve describes combinations of risk and return that leaves the investor equally satisfied.

For a risk-averse person, these ICs will be upward sloping because risk is undesirable. Thus with a greater amount of risk, it takes a greater amount of return to make the individual equally well off.

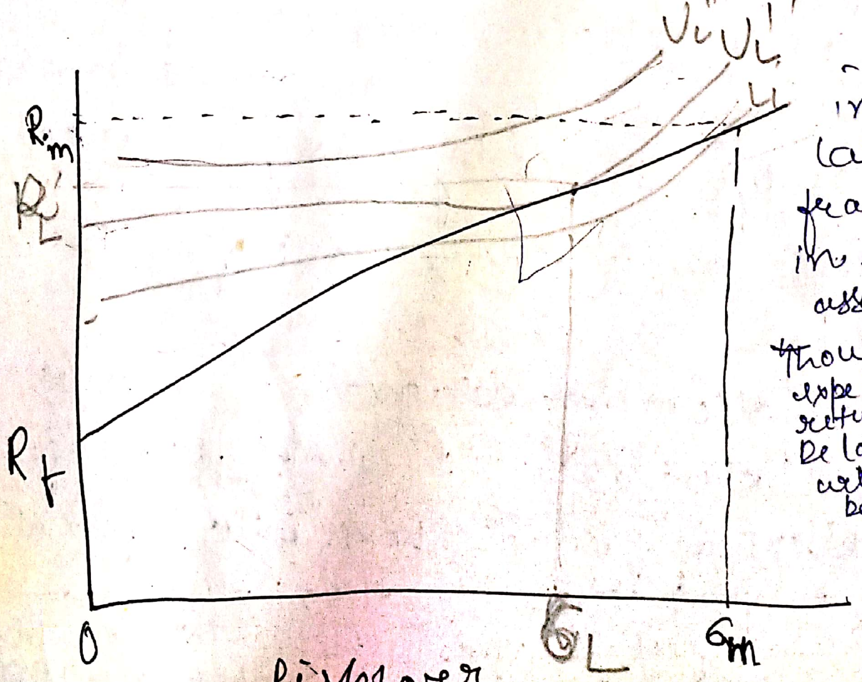
~~~~ But for a risk lover, IC's will be comparatively flatter because he ~~~~ is willing to take higher risks for a slightly higher expected return.

Eqm is attained where budget line is tangent to IC.

The investor will try to reach higher IC because for a given risk, return is higher on a higher IC.



For a risk averse person, portfolio consist of mainly risk free asset. Thus expected return R_A' slightly greater than risk-free return, σ_A' is small.



invests large fraction in risky asset, though expected return will be larger, it will also be riskier.

Risk Lover
~~~~

10 Q) ① Define Adverse selection [2]
Distinguish between adverse selection and moral hazard. [4]

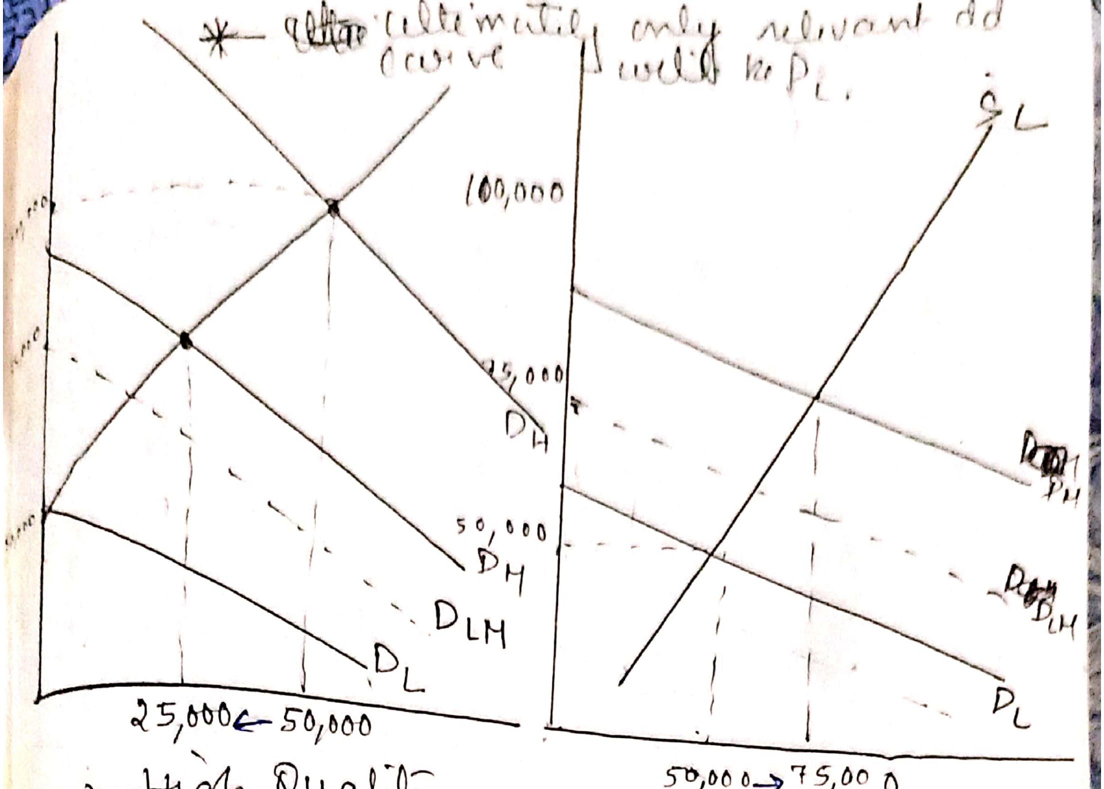
Ans When there is asymmetric information – the situation in which a buyer and a seller possess different information about a transaction, adverse selection occurs.

Adverse selection is a form of market failure resulting when products of different qualities are sold at a single price because of asymmetric information, so that too much of the low-quality product and too little of the high quality are sold.

Moral Hazard :- when a party whose actions are unobserved can affect the probability or magnitude of a payment associated with an event.

We can explain the concept of Adverse selection with an example of analysis made by George Akerlof on market for lemon — mkt for used car.

The following diagram shows that when there is imperfect information here sellers of products have better information about product quality than buyers — thus 'lemons problem' arises in which low quality goods are driven out by high quality goods.



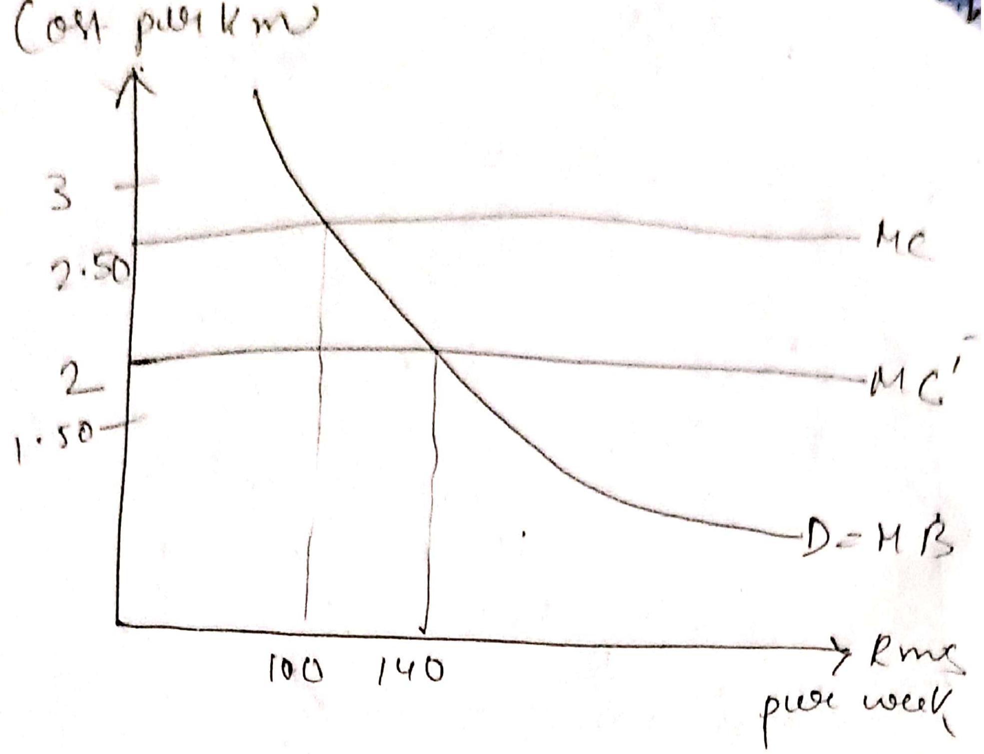
(a) High Quality cars

(b) Low-Quality cars.

D_H & D_L are dd curves for high quality and low quality cars respectively.

Since initially buyer views 50,000 of each kind of car is sold, so while making a purchase, buyer views all cars as of "medium" quality (50% of both) so this perceived demand shifts to D_M, Likewise in (b), perceived dd curve for low quality cars shifts from D_L to D_M. As a result sale of high quality car falls & that of low quality rises. As consumers realize that most cars sold are of low quality, again the perceived demand shifts to D_{LM}, thus more of low quality is sold. This shifting (as high quality falls) continues until only low-quality cars are sold. Mkt price too low to bring any high quality cars.

* Low quality goods can drive high quality goods out of mkt.



Moral hazard alters the ability of markets to allocate resources efficiently. D gives the demand for automobile driving [downward sloping; people switch to alternative transport as cost increases]. With no moral hazard the driver drives 100 miles, which is efficient amount. With moral hazard, the driver perceives the cost per mile to MC=2, & drives 140 miles.

Moral hazard, not only alters behaviour, but also creates economic inefficiency bcz insured individual perceives either the cost or benefit of activity differently from true social cost or benefit.

From Maddala

for Adverse selection:- In case of ~~lemons~~ mkt for used cars, sellers of lemons will get more than what their cars are worth and sellers of good - quality cars will get less than what their cars are worth because of imperfect information.

Thus average quality goes down and so does the average price buyers are willing to pay. Again, cars with better-than-average quality go off the market and the process continues until 'lemons' drive out all good cars from mkt.

The problem here is that buyers and sellers are unable to communicate accurate information. However, in practice, numerous institutions arise that counteract the adverse effects of quality uncertainty suggested by lemons model. For ex - there are automotive service centres that can check the quality of used cars and customers can get some information on quality at a cost. Thus information also bears a cost.

Moral hazard :-

Adverse selection - Even a seller ~~would~~ who ~~expects to sell~~ have a brand new car, which he know is in perfect condition expects to sells it in much less than he paid for it.

Paper CC 202:

Managerial Economics (ME)

Module-II

Unit 6: Managerial Theories of Firm

[Prepared by Dr. Susmita Chatterjee & Dr. Nilanja Biswas]

WILLIAMSON'S THEORY OF MANAGERIAL UTILITY MAXIMISATION

Managerial utility maximisation theory, developed by American economist Oliver E Williamson, describes managers' utility versus profit maximisation in corporate environment, where management is separated from owners (shareholders). According to the theory managers take decisions that prioritise their own utility maximisation over principals' profits, provided the firm can generate minimum profit demanded by the principals to maintain managers' job security. In this theory the principal agent problem emerges.

The managerial utility function includes variables such as salary, job security, power, status, dominance, prestige and professional excellence of managers. Of these, salary is the only quantitative variable and thus measurable. The other variables are non-pecuniary, which are non-quantifiable. The variables expenditure on staff salary, management slack, discretionary investments can be assigned nominal values. Thus these will be used as proxy variables to measure the real or unquantifiable concepts like job security, power, status, dominance, prestige and professional excellence of managers, appearing in the managerial utility function.

Utility function or **"expense preference"**]of a manager can be given by:

$$U=U(S,M,I_d)$$

where *U* denotes the Utility function, *S* denotes the “monetary expenditure on the staff”, *M* stands for "Management Slack" and I_D stands for amount of "Discretionary Investment".

"**Monetary expenditure on staff**" include not only the manager's salary and other forms of monetary compensation received by him from the business firm but also the number of staff under the control of the manager as there is a close positive relationship between the number of staff and the manager's salary.

"**Management slack**" consists of those non-essential management perquisites such as entertainment expenses, lavishly furnished offices, luxurious cars, large expense accounts, etc. which are above minimum to retain the managers in the firm. These perks, even if not provided would not make the manager quit his job, but these are incentives which enhance their prestige and status in the organisation in turn contributing to efficiency of the firm's operations. The Management Slack is also a part of the cost of production of the firm.

"**Discretionary investment**" refers to the amount of resources left at a manager's disposal, to be able to spend at his own discretion. For example, spending on latest equipment, furniture, decoration material, etc. It satisfies their ego and gives them a sense of pride. These give a boost to the manager's esteem and status in the organisation. Such investments are over and above the amount required for the survival of the firm (such as periodic replacement of the capital equipment).

Concepts of profit in the model

Williamson has put forth four main concepts of profits in his model:

Actual profit (Π)

$$\Pi = R - C - S$$

where *R* is the total revenue, *C* is the cost of production and *S* is the staff expenditure.

Reported profit (Π_r)

$$\Pi_r = \Pi - M$$

where Π is the actual profit and *M* is the management slack.

Minimum profit (Π_0)

It is the amount of profit after tax deducted which should be paid to the shareholders of the firm, in the form of dividends, to keep them satisfied. If the minimum level of profit cannot be given out to the shareholders, they might resort of bulk sale of their shares which will transfer the ownership to other hands leaving the company in the risk of a complete take over. Since the shareholders have the voting rights, they might also vote for the change of the top level of management. Thus the job security of the manager is also threatened.Ideally the reported profits must be either equal to or greater than the minimum profits plus the taxes, as it is only after paying out the minimum profit that the additional profit can be used to increase the managerial utility further.

$$\Pi_r \geq \Pi_0 + T$$

where Π_r is the reported profit, Π_0 is the minimum profit and *T* is the tax.

Discretionary profit (Π_D)

It is basically the entire amount of profit left after minimum profits and tax which is used to increase the manager's utility, that is, to pay out managerial emoluments as well as allow them to make discretionary investments.

$$\Pi_D = \Pi - \Pi_0 - T$$

where Π_D is the discretionary profit, Π is the actual profit, Π_0 is the minimum profit and *T* is the tax amount.

However, what appears in the managerial utility function is discretionary investments (I_D) and not discretionary profits. Thus it is very important to distinguish between the two as further in the model we would have to maximize the managerial utility function given the profit constraint.

$$I_D = \Pi_r - \Pi_0 - T$$

where Π_r is the reported profit, Π_0 is the minimum profit and *T* is the tax amount.

Thus it can be seen that the difference in the Discretionary Profit and the Discretionary investment arises because of the amount of managerial slack. This can be represented by the given equation

$$\Pi_D = I_D + M$$

where Π_D is the discretionary profit, I_D is the Discretionary investment and M is the management slack.

Model framework

For simple representation of the model the managerial slack is considered to be zero. Thus there is no difference between the actual profit and reported profit, which implies that the discretionary profit is equal to the discretionary investment. I.e.

$\Pi_r = \Pi$ or $\Pi_D = I_D$

where Π_r is the reported profit, Π is the actual profit, Π_D is the discretionary profit and I_D is the discretionary investment.

Such that the utility function of the manager becomes

$U=U(S, I_D)$

where **S** is the staff expenditure and $\mathbf{I_D}$ is the discretionary investment.

There is a trade off between these two variables. Increase in either will give the manager a higher level of satisfaction. At any point of time the amount of both these variables combined is the same, therefore an increase in one would automatically require a decrease in the other. The manager therefore has to make a choice of the correct combination of these two variables to attain a certain level of desired utility.

Substituting $\boldsymbol{I_D = \Pi - \Pi_0 - T}$

in the new managerial utility function, it can be rewritten as

$$U=U\ (s,\ \Pi-\ \Pi_0\ -T)$$

The relationship between the two variables in the manager's utility function is determined by the profit function. Profit of a firm is dependent on the demand and cost conditions. Given the cost conditions the demand is dependent of the price, staff expenditures and the market condition.

$$X=f(S,\bar{P},\bar{E})$$

Price and market condition is assumed to be given exogenously at equilibrium. Thus the profit of the firm becomes dependent on the staff expenditure which can be written as

$$\Pi=f\ (X)=f(S,\bar{P},\bar{E})$$

Discretionary profit can be rewritten as

$$\Pi_D = f(S,\bar{P},\bar{E})-\ \Pi_0 - T)$$

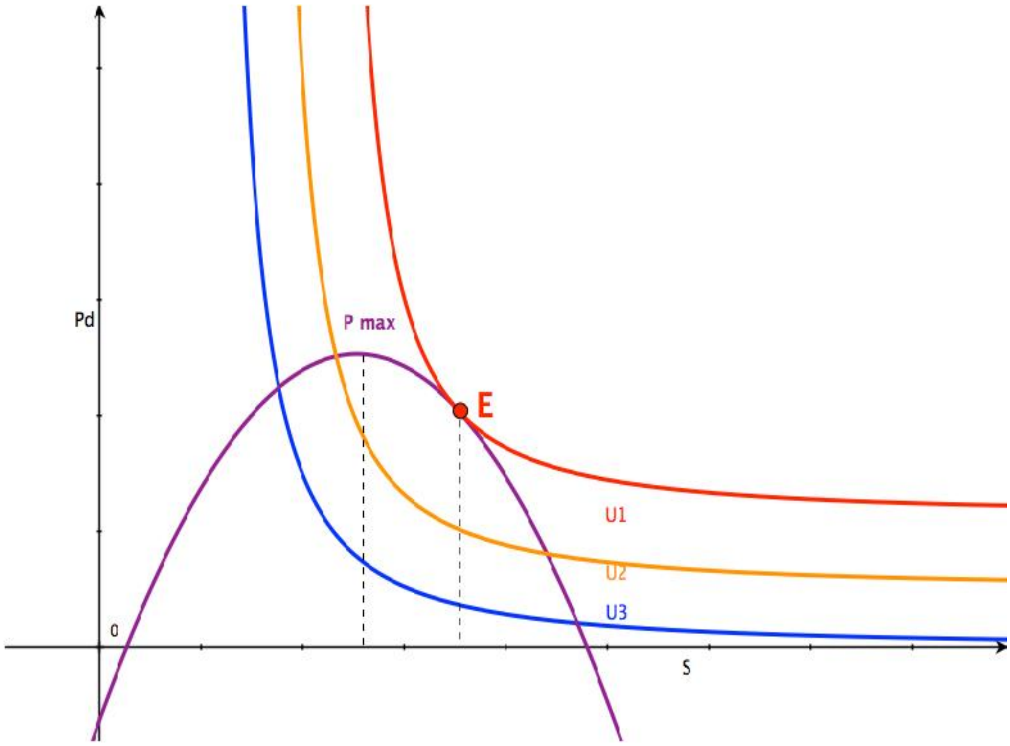
In the model, the managers would try to maximise their utility given the profit constraint

$$\text{Max } U=U\ (S,\ \Pi-\ \Pi_0\ -T)$$

$$\text{Subject to } \Pi \geq \Pi_0 + T$$

In the graph below U1 corresponds to the highest level of Utility on which either combination of Staff Expenditure and Discretionary Profit give manager the same satisfaction.

Graphical Representation



A manager would aim to achieve combination of Staff Expenditure and Discretionary Profit at the level of utility U1, however in the point of equilibrium with firms profit (above minimum profit) the level of Staff Expenditure is such that Discretionary Profit is not at the maximum and therefore shareholders are not earning all the profit they could if manager's utility level was lowered through decreased Staff Expenditure and/or Management Slack. Has the manager allocated resources so that his utility would be at lower levels U2 or U3 then Discretionary Profit and therefore overall Profit of the firm would be higher.

In other words, once minimum profit has been achieved managers would tend to allocate firm's resources towards increasing their own utility rather than maximising shareholder's profit.

Marris Model of Managerial Enterprise

R Marris had developed 'A Model of Managerial Enterprise' where the goal of the firm is maximixation of balanced rate of growth (g*), growth of demand for the products of the firm (g_D) and growth of supply of capital (g_C). According to Marris by jointly maximizing the rate of growth of demand of the products and capital the managers achieve maximization of their own utility as well as the utility of the owners-shareholders.

The utility of the managers is U_M =f (salaries, power status, job security)

The utility of the owners is U_O =f (profits, capital, output, market share, public esteem)

Since Marris restricts his model to steady rate of growth over time it can be argued that most of the relevant economic magnitudes change simultaneously with the long run growth rate. Hence the Utility function of the managers and owners can be written as

U_M =f ($g_{D,}$ s), U_O =f(g_C) where 's' is a measure of the job security.

Now s can be measured be a weighted average of three crucial ratios namely liquidity ratio, leverage-debt ratio and the profit retention ratio. The three ratios reflect the financial policy of the firm and are given as

Liquidity Ratio = Liquid Assets / Total Assets = L/A = a_1

Leverage or debt ratio = Value of Debts/ Total Assets = D/A = a_2

Retention Ratio = Retained Profits/ Total Profit = a_3

The three financial ratios are combined into a single financial parameter (a) which is known as the 'financial security constraint'. This is weighted average of the three above ratios.

Given the objective the constraints faced by the managers are

a) Managerial Constraint – Marris followed Penrose's concept of a constraint set by the capacity of the top management. The managerial capacity can be increased by hiring new mamagers.
b) Job Security Constraint- this is a constraint set by the managers to the disutility associated with the dismissal from job. The constraint is influenced by the financial policy of the firm which again is controlled by the optimal levels of financial ratios.

Model: The objective of the firm is to maximise the balanced growth rate (g*) where the instrumental variables are a (financial constraints), d (rate of diversification) and m (profit margin).

According to the Average pricing rule

P=C+A+(R&D) + m

Where P= price given from the market

C= Production Cost(given)

A= Advertising cost and other selling expense

R & D = Research and Development expense

m= average profit margin.

m= P-C-A-(R&D) and $\frac{\partial m}{\partial A} < 0, \frac{\partial m}{\partial (R\&D)} < 0$

further m is used as a proxy for the policy variables A and R&D. Given this pricing rule the firm maximizes the balanced rate of growth.

Now, g_D = f(d,k) where d=diversification rate defined as the number of new products introduced per time period, k= proportion of successful new products.

But according to Marris Diversification can be of two forms: a) Differentiated Diversification- Firm introduces a completely new product which has no close substitutes. b) Imitative Diversification- Firm introduces a new product which is a substitute for similar commodities already produced by existing competitors.

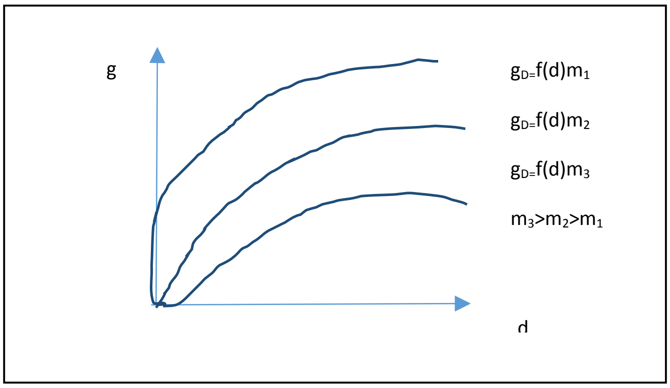
It is assumed that g_D is positively related to d but g_D increases at a diminishing rate with d, $\frac{\partial gD}{\partial d} > 0.$

Again k= f(d, P, A , R & D, intrinsic value)

Since m is used as a proxy for the policy variables A and R&D and m is negatively related to A and R&D so it can be inferred that k is negatively related to m.

Thus summarising it can be written g_D= f(d,m) where $\frac{\partial gD}{\partial d} > 0, \frac{\partial gD}{\partial m} < 0$

Graphically this can be shown as



The above diagram imply that at a given price of the product lower m implies

a) Larger A and or R&D expenses
b) Larger k
c) Higher g_D

According to the assumptionsof the Marris model the rate of growth of capital supply g_C is given as

g_C=a.f(π)

a= financial security constraint, π= level of total profits.

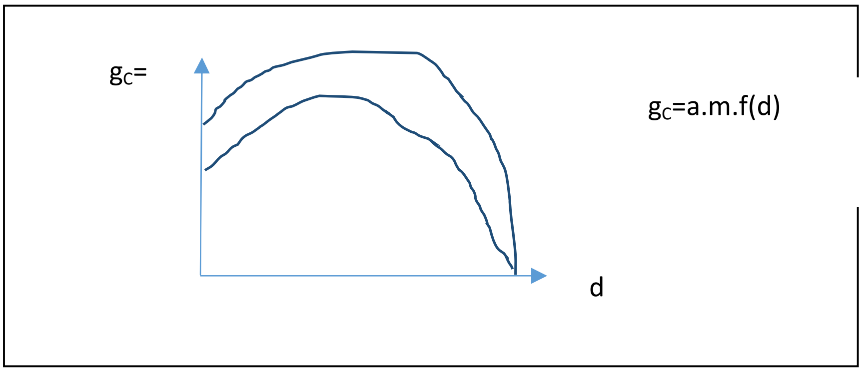
Now, π=(m, k/X) where m= average rate of π, k/X= capital output ratio.

Now, $\frac{\partial \pi}{\partial m} > 0$, and k/X=f(d) where given k the relation between d and X is as follows:

a) Upto a certain level of d, 'X' and 'd' are +ly related.
b) Beyond that X reaches a maximum
c) Then X falls with further increase in d.

Substituting k/X in the profit function π=(m, d)

Using this it can be written g_C= a.f(m, d) where according to Marris a the financial parameter is exogenously determined by the risk attitudes of the managers. Then keeping 'a' fixed there is a +ve relation between g_C and 'm'. Lastly keeping 'a' and 'm' constant as 'd' increases (upto the point of optimal use of R&D and team of managers) then after that g_C is –vely related with 'd'. Thus the following curve is drawn

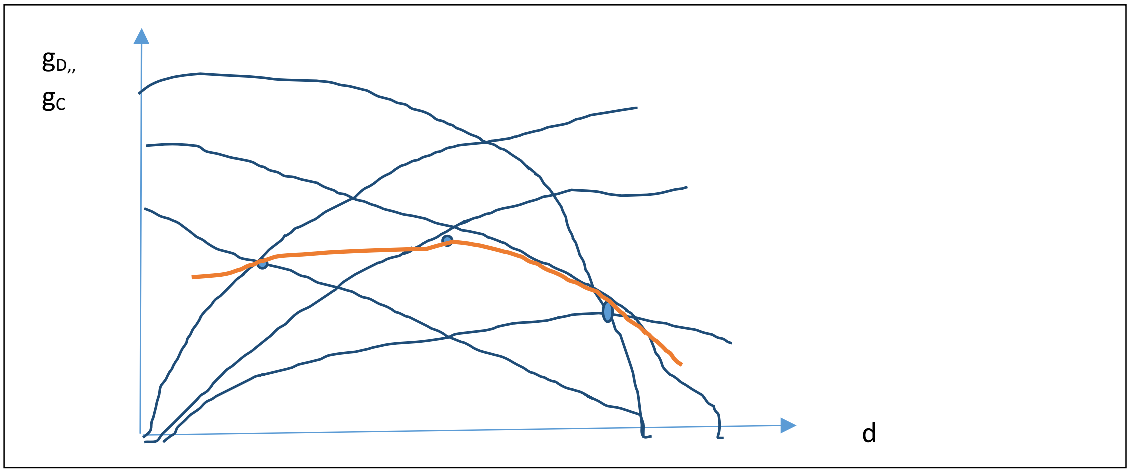


Summarizing the above arguments the Marris Model in its complete form can be given as:

1. g_D=f(m,d) Demand –Growth equation
2. π=(m, d) Profit equation
3. g_C= a.f(m, d) supply of capital equation
4. a ≤ a* security constraint
5. g_D=$g_C$balanced growth equilibrium condition

In the above equation structure 'a' is exogenously determined, π is endogenously determined, 'm' and 'd' are policy instruments. From the balanced growth equilibrium condition there is one equation with two unknowns 'd' and 'm'. The model can only be solved if one of the variables 'm' or 'd' is subjectively determined by the managers.

Graphically the equilibrium of the firm can be represented by the intersection of the $g_D$and $g_C$curves associated with a given profit rate.



The equilibrium

Marris defines the curve joining the intersection points as the balanced growth curve given the financial constraint ‘a’. The firm is in equilibrium when it reaches the highest point on the balanced growth curve.

Cyert and March Model

The long-established Microeconomic theory of firm behaviour considers profit maximization to be the sole goal of a firm. It is considered that though a firm can face alternate markets viz., perfect competition, monopolistic competition, oligopoly or monopoly, but in every case the owners will seek to maximize the profits only. In contrast the managerial theories of the firm (or organization) consider the firm to be a coalition. Firm is not owned by a single owner, rather by multiple stockholders. The interests of these stockholders, managers, workers, consumers and suppliers may differ. The focus of behavioural model of Cyert and March is to explain the methods of conflict resolution.

The firm which is being considered in this model is mainly a large multiproduct firm owned by a number of stockholders. Owners of firm and managers of firm are two different groups. Firm is generally considered to operate in imperfectly competitive market. As information is not complete, uncertainty prevails. This type of assumption about a firm is more realistic. Each group within a firm, be it owners, managers, workers, consumers and suppliers, will have different utility functions. Owners will seek to maximize profits, managers will seek better salaries and power, workers will seek higher wages and better conditions to work, consumers will seek low price but more variety and quality and so on. But out of all these groups, the most important group is considered to be 'managers', followed by shareholders and workers.

The most important feature of the behavioural theory is that it does not focus directly on the goals of a multiproduct large firm rather tries to explain the process i.e., 'How these goals originate?' According to Cyert and March, the goals of firms originate mainly because of demands of alternate groups. These demands further depend on availability of information, expectations, aspirations and achievements of other groups in same or other firms. The basic dichotomy in the structure of firm is accepted in the behavioural model. In this dichotomy, on one hand there is the organization (as a whole) called as 'firm' and on other hand, there are the individual groups and sub-groups within the firm. The individual groups and sub-groups within the firm will have different objectives than 'firm' as a whole. Further, it is considered that demands and past achievements are highly correlated. Demands of each group do not remain static and as keep on changing according to past achievements (of group and that of other groups, in getting their demands met) and other changes in the firm and its environment. Here changes in firm are very significant because if firm performance remains static or stagnant, demands may also remain static.

ROLE OF TIME-LAG : By time lag we mean the lag between past achievements and future aspirations. According to Cyert and March, this time lag can be used by the firm to generate and accumulate surplus which eventually can be used for conflict resolution (conflict between different groups as the demands of various groups and sub-groups may be in continuous conflict with each other and the groups may be continuously bargaining with each other).

ROLE OF TOP MANAGEMENT: The role of top management is extremely significant in Cyert and March model. Not only the top management sets the goals of the firm, as the goals may be in conflict with various groups within the firm, top management works towards reconciliation of goals too.

According to Cyert and March, a firm has mainly five goals:

- How much to produce (Production goal)
- Inventory goal
- Sales goal
- Market share goal
- Profit goal

1) Production Goal : Mainly it is the production department which takes care of production goal. Smooth production process also implies that production is evenly distributed over time and seasonal as well as cyclical variations in demand are taken care of. If demand is too high, it may require overworking by workers and other factors of production. Similarly, if demand dips, it may lead to over-production and lay-off of workers.

2) Inventory Goal This goal either may come from production department or from sales department. In some instances the firm may have a separate inventory department too. Production department will always seek sufficient stock of raw materials while the sales department will seek sufficient stock of finished product.

3) Sales Goal The strategy for sales will be a part of sales goal.

4) Market Share Goal It may further involve market research, analyzing the competitors and deciding the advertisement strategy.

5) Profit Goal Top management sets the profit goal to satisfy the shareholders. Furthermore, as the firm may have relied on banks and other financial institutions for its financing, profit goal also acts as a benchmark to satisfy them.

According to Cyert and March, law of diminishing returns operates even in case on top managers' abilities to take decisions. Therefore as the goals increase, the efficiency of decision making may decrease. Therefore, the firms mainly focus on satisfying behaviour. The concept of ‘Satisfying behaviour’ was originally given by Simon (1955). He writes, “ Among the common constraints- which are not themselves the objects of rational calculation-are (1) the set of alternatives open to choice, (2) the relationships that determine the pay-offs(“satisfactions” “goal attainment”) as a function of the alternative that is chosen, and (3)the preference ordering among pay-offs. The selection of particular constraints and the rejection of others for incorporation in the model of rational behaviour involves implicit assumptions as to what variables the rational organism “controls”-and hence can “optimize” as means to rational adaptation-and what variables it must take as fixed.

MEANS FOR CONFLICT RESOLUTION : As various groups compete with each other for their individual group-specific goals, there is continuous struggle and bargaining within the firm. Interestingly, this does not lead to chaos within the firm and firm keeps on performing in stable manner as before.

The reasons for stability can be listed as:

The groups have limited bargaining time and may not be aware/examine all the alternatives open to them. The budget share allotted to each department also acts as a constraint. There may be penalty involved with underutilization or over-utilization of the budget. Past history of goal setting and achieving/not achieving the goals.

Clear delegation of authority also minimizes conflicts. The main means or channels used for conflict resolution are: Cash payments: The managers and workers receive cash salary, owners receive¬ dividends etc. Incentives: In behavioural theory incentives imply side payments (like funding a¬ research project, if a scientist within the firm needs to be incentivized).

Slack payments: Organizational slack consists in payments to the members of the¬ coalition in excess of what is required to maintain the organization. According to behavioural theory, this slack may be earned by every group. The managers may receive it in form of fancy offices and cars, workers may receive in in form of wages higher than market wages, customers may receive it in terms of discounts and shareholders may receive it in terms of higher dividends. All this is possible only if business period is favourable (so that sufficient surplus is generated), and there is sufficient time lag available between aspirations. Sequential attention to demand: The most important aspect of conflict resolution is¬ sequential attention to demand i.e., demands of various groups be met priority wise. In peek production period, production department will get priority; but if focus is on sale/market share, the priority may change accordingly.

DECISION-MAKING AT TOP MANAGEMENT LEVEL : Budget allocation according to bargaining skills. Three things are taken into consideration while allotting the budget Goals of the firm

¬ Availability of resources

¬ Bargaining skills of head of each department Though, top management does keep some funds aside. These funds can be used at its own discretion at any stage as per requirement.

-The decisions by top management are based on bounded rationality. Complete information may not be sought (as information is not free). Collecting information about all the alternatives can be uneconomical time-wise too. As no detailed costbenefit analysis is undertaken, one can conclude that top management acts in 'limited' rational way. Information is generally searched only if some problem is there. Here, the concept of position bias comes in the fore. The desire of various managers for security and power in the organization leads to this bias. Just to show importance of their demand, they may overstate the requirements and this may eventually lead to an upward bias in the cost structure of the firm. Furthermore, information may be distorted or under-reported.

DECISION-MAKING AT LOWER MANAGEMENT LEVEL: The day to day routine decisions are taken in a simplified manner by the established norms and rules within the firm. The administrative staff follows the principle of 'learning-by-doing' and learns from past successes and mistakes. Failed methods are not repeated.

UNCERTAINITIES FACED BY FIRM: Cyert and March explain two types of uncertainties faced by a firm, (1) The market uncertainty and (2) the uncertainty about competitors' reaction. The market uncertainty can be partly avoided by collecting information. Because of market uncertainty, firm mainly depends on short term decisions and avoids long term decisions. Interestingly, in terms of uncertainty arising because of oligopolistic interdependence, the behavioural theory chooses to be quiet by assuming 'tacit collusion' among existing firms.